The Melitz Model: Slides

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Empirical Evidence

BEJK (2003)

- Exporters are in the minority. In 1992, only 21% of U.S. plants reported exporting anything.
- Exporters sell most of their output domestically: around 2/3 of exporters sell less than 10% of their output abroad.
- Exporters are bigger than non-exporters: they ship on average 5.6 times more than nonexporters (4.8 times more domestically).
- Plants are also heterogeneous in measured productivity.
  - exporters have, on average, a 33% advantage in labor productivity relative to nonexporters

This suggests that the most productive firms self-select into export markets, but it could also reflect learning by exporting.

Empirical Evidence

- There are substantial reallocation effects within an industry following trade liberalization episodes.
  - Exposure to trade forces the least productive firms to exit or shut-down (Bernard and Jensen (1999), Aw et al. (2000), Clerides et al. (1998)).
  - Trade liberalization leads to market share reallocations towards more productive firms, thereby increasing aggregate productivity (Pavcnik (2002), Bernard and Jensen (1999b)).
- Theoretical frameworks for studying firms and the decision to export should include two features:
  - within sectoral heterogeneity in size and productivity
  - a feature that leads only the most productive firms to engage in foreign trade

Motivation of Melitz (2003)

Melitz (2003) develops a *highly tractable* framework that captures the empirical evidence.

Notice! The Krugman model is not able to explain the evidence discussed above. In the model, all firms are identical and export to all possible destinations.

Key Features

- The Krugman model (with monopolistic competition and increasing returns) with heterogenous firms.
- Variable and fixed costs of trade.
- BEJK (2003) develops another approach based on the Ricardian model of trade that is also able to capture the evidence!
  - fixed (exogenous) number of varieties: competition between domestic and foreign producers of the same variety
  - endogenous markups

- A CES utility function over a continuum of goods indexed by ω:

$$Q = \left[\int_{\omega \in \Omega} q(\omega)^{\rho} d\omega \right]^{1/\rho}$$

where Ω is the set of available goods and $\rho \in (0, 1)$. The elasticity of substitution between two goods is

$$\sigma = \frac{1}{1-\rho} \iff \rho = \frac{\sigma - 1}{\sigma}.$$

- Demand for a certain variety:

$$q(\omega) = Q \left[\frac{p(\omega)}{P} \right]^{-\sigma}.$$

- The CES price index:

$$P = \left[\int_{\omega \in \Omega} p(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}$$

- Expenditure on a variety ω:

$$r(\omega) = p(\omega)q(\omega) = R \left[\frac{p(\omega)}{P} \right]^{1-\sigma},$$

where $R = PQ = \int_{\omega \in \Omega} r(\omega) d\omega$ is aggregate expenditure.

Model: Closed Economy

Production

- Continuum of firms
- Labor is the only factor of production: L
- Total costs:

$$l = f + q/\varphi$$

  where φ is a firm specific productivity and q is the output.

- Given the demand function:

$$p(\varphi) = \frac{w}{\rho\varphi}$$

  where w is the wage rate, which is normalized to unity: $w = 1$.

Model: Closed Economy

Production

- Firm profit is given by

$$\pi(\varphi) = \frac{R}{\sigma} (P\rho\varphi)^{\sigma-1} - f$$

- Revenues:

$$r(\varphi) = p(\varphi)q(\varphi) = R (P\rho\varphi)^{\sigma-1}.$$

- Notice that

$$\pi(\varphi) = \frac{r(\varphi)}{\sigma} - f.$$

- Finally, for any φ_1 and φ_2,

$$\frac{r(\varphi_1)}{r(\varphi_2)} = \left(\frac{\varphi_1}{\varphi_2}\right)^{\sigma-1}.$$

- In summary, a more productive firm will be bigger (larger output and revenues), charge a lower price, and earn higher profits than a less productive firm.

- An equilibrium is characterized by a mass M of firms (and hence M goods) and a distribution $\mu(\varphi)$ of productivity levels over a subset of $(0, \infty)$.
- You can think that there are $M\mu(\varphi)$ firms with productivity φ.
- Then, the price index

$$
\begin{aligned}
P &= \left[\int_0^\infty p(\varphi)^{1-\sigma} M\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= \left[\int_0^\infty \left[\frac{1}{\rho\varphi}\right]^{1-\sigma} M\mu(\varphi)d\varphi\right]^{\frac{1}{1-\sigma}} \\
&= M^{\frac{1}{1-\sigma}} \frac{1}{\rho\left[\int_0^\infty \varphi^{\sigma-1}\mu(\varphi)d\varphi\right]^{\frac{1}{\sigma-1}}}.
\end{aligned}
$$

Model: Closed Economy

Aggregation

- Let us define

$$\tilde{\varphi} = \left[\int_0^\infty \varphi^{\sigma-1} \mu(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}}$$

  as a weighted average of the firm productivities.

- Then,

$$P = M^{\frac{1}{1-\sigma}} p(\tilde{\varphi})$$

$$Q = M^{1/\rho} q(\tilde{\varphi})$$

- Moreover,

$$
\begin{aligned}
R &= PQ = Mp(\tilde{\varphi})q(\tilde{\varphi}) = Mr(\tilde{\varphi}) \\
\Pi &= M\pi(\tilde{\varphi})
\end{aligned}
$$

  where $\Pi = \int_0^\infty \pi(\varphi)M\mu(\varphi)d\varphi$ is total profits.

- An industry comprised of M firms with any distribution of productivity levels $\mu(\varphi)$ that yields the same average productivity $\tilde{\varphi}$ will also induce the same aggregate outcome as an industry with M representative firms sharing the same productivity $\varphi = \tilde{\varphi}$ (the Krugman model).

Model: Closed Economy

Firm Entry and Exit

- There is a large (unbounded) pool of prospective entrants into the industry.
- To enter, firms must make an initial investment modeled as a fixed entry cost $f_e > 0$ (measured in labor units).
- Firms then draw their initial productivity parameter φ from a common distribution $g(\varphi)$ with the support on $[0, \infty)$.
- Conditional on the productivity drawn, firms decided whether to stay and produce or to exit and not produce.
- If the firm does produce, it then faces a constant probability δ in every period of a bad shock that would force it to exit.
- We consider only steady state equilibria in which aggregate variables remain constant over time.

- Each firm's value function is given by

$$v(\varphi) = \max(0, \sum_{t=0}^{\infty}(1-\delta)^t \pi(\varphi)) = \max(0, \frac{\pi(\varphi)}{\delta}).$$

  Notice that φ does not change over time.

- Define φ^* as the cutoff level of productivity. That is, firms with $\varphi < \varphi^*$ exit and do not produce. Then,

$$\varphi^* = \inf\left(\varphi : v(\varphi) > 0\right).$$

- The zero profit condition:

$$\pi(\varphi^*) = 0.$$

Model: Closed Economy

Firm Entry and Exit

- Therefore, $\mu(\varphi)$ is the conditional distribution of $g(\varphi)$ on $[\varphi^*, \infty)$:

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)}, & \text{if } \varphi \geq \varphi^* \\ 0, & \text{otherwise,} \end{cases}$$

  and $p_{in} = 1 - G(\varphi^*)$ is the probability of successful entry.

- Therefore,

$$\tilde{\varphi} = \tilde{\varphi}(\varphi^*) = \left[\frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \varphi^{\sigma-1} g(\varphi) d\varphi \right]^{\frac{1}{\sigma-1}} .$$

Model: Closed Economy

Zero Cutoff Profit Condition

- Let us denote $\bar{r}$ as average revenues (revenues per firm), then

$$\bar{r} = \frac{R}{M} = r(\tilde{\varphi}).$$

- Moreover, we have

$$\frac{r(\tilde{\varphi})}{r(\varphi^*)} = \left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1}.$$

- Hence,

$$\bar{r} = \left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} r(\varphi^*).$$

- In the same manner, $\bar{\pi}$ is denoted as average profits (profits per firm). That is,

$$\bar{\pi} = \frac{\Pi}{M} = \pi(\tilde{\varphi}) = \frac{r(\tilde{\varphi})}{\sigma} - f$$

- Thus, we obtain that

$$\bar{\pi} = \left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} \frac{r(\varphi^*)}{\sigma} - f.$$

Model: Closed Economy

Zero Cutoff Profit Condition

- The zero profit condition means that

$$\begin{aligned} \pi(\varphi^*) &= \frac{r(\varphi^*)}{\sigma} - f = 0 \iff \\ r(\varphi^*) &= \sigma f. \end{aligned}$$

- Therefore,

$$\begin{aligned} \bar{\pi} &= \left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} \frac{r(\varphi^*)}{\sigma} - f \\ &= \left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} \frac{\sigma f}{\sigma} - f \\ &= f\left(\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right). \end{aligned}$$

Model: Closed Economy

Free Entry

- The net value of entry:

$$
\begin{aligned}
v_e &= (1 - G(\varphi^*)) \sum_{t=0}^{\infty} (1-\delta)^t \bar{\pi} - f_e \\
&= (1 - G(\varphi^*)) \frac{\bar{\pi}}{\delta} - f_e
\end{aligned}
$$

- Free entry implies that

$$
\begin{aligned}
v_e &= 0 \iff \\
\bar{\pi} &= \frac{\delta f_e}{1 - G(\varphi^*)}.
\end{aligned}
$$

- We have

$$\bar{\pi} = f\left(\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right)$$

$$\bar{\pi} = \frac{\delta f_e}{1 - G(\varphi^*)}.$$

- So, there are two equations and two unknowns and we can solve for φ^* and $\bar{\pi}$.
- If we know φ^*, we can find $\tilde{\varphi}$, as $\tilde{\varphi} = \tilde{\varphi}(\varphi^*)$.

Model: Closed Economy

Equilibrium: Mass of Firms

- Let us denote M_e as the mass of entrants in each period.
- In steady state:

$$(1 - G(\varphi^*))\, M_e = \delta M.$$

- Labor market clearing condition:

$$L = L_p + L_e$$

  where L_p is labor used in production and L_e is labor used to enter the industry.

- Moreover,

$$L_p = R - \Pi.$$

- Finally,

$$L_e = M_e f_e.$$

- Therefore,

$$L_e = M_e f_e = \frac{\delta M}{1 - G(\varphi^*)} f_e = M \bar{\pi} = \Pi$$

- In addition,

$$R = L_p + \Pi = L_p + L_e = L.$$

- Hence,

$$
\begin{aligned}
R &= L \\
M &= \frac{R}{\bar{r}} = \frac{L}{\sigma(\bar{\pi} + f)}.
\end{aligned}
$$

- Finally, the price index is given by

$$
P = M^{\frac{1}{1-\sigma}} p(\tilde{\varphi}) = M^{\frac{1}{1-\sigma}} \frac{1}{\rho \tilde{\varphi}}.
$$

- Notice that all the firm-level variables (φ^*, $\tilde{\varphi}$, $\bar{\pi}$, and $\bar{r}$) do no depend on the country size L.
- Welfare per worker:

$$W = \frac{w}{P} = M^{\frac{1}{\sigma-1}} \rho \tilde{\varphi}.$$

- So, we have the variety effect (through M) and the productivity effect (through $\tilde{\varphi}$).
- Welfare in a larger country is higher due to only the variety effect (a la Krugman (1980)).
- Note that in Krugman (1980), we can derive a similar expression for welfare, if all firms have productivity $\tilde{\varphi}$. However, in the Krugman model, $\tilde{\varphi}$ would be exogenous and is not affected by trade. In this model, $\tilde{\varphi}$ is an endogenous variable and is affected by trade.

Model: Open Economy

- If there are no trade costs, then trade allows the individual countries to replicate the outcome of the integrated world equilibrium. It is equivalent to a rise in a country size L. In this case, there is no effect on the firm-level variables and welfare increases because of the variety effect. The same result can be derived in the Krugman model.
- In the model, there are both variable (iceberg) and fixed costs of trade. Fixed costs should be paid after the realization of productivity. That is, firms know φ and then decide whether to export or not.
- $n+1$ identical countries. Therefore, the wage is the same in all countries (FPE) and is normalized to unity.

Model: Open Economy

Pricing Rule

- The domestic (producer) price is

$$p_d(\varphi) = \frac{1}{\rho\varphi}.$$

- The export price is given by

$$p_x(\varphi) = \tau p_d(\varphi) = \frac{\tau}{\rho\varphi}$$

where τ is the Iceberg transport costs.

Model: Open Economy

Revenues

- The revenues earned from domestic sales are given by

$$r_d(\varphi) = R\left(P\rho\varphi\right)^{\sigma-1}$$

- Revenues from export sales:

$$\begin{aligned} r_x(\varphi) &= R\left(P\frac{\rho\varphi}{\tau}\right)^{\sigma-1} \\ &= \tau^{1-\sigma}R\left(P\rho\varphi\right)^{\sigma-1} \\ &= \tau^{1-\sigma}r_d(\varphi). \end{aligned}$$

- The total revenues depend on the export status:

$$r(\varphi) = \begin{cases} r_d(\varphi), & \text{if the firm does not export} \\ r_d(\varphi) + nr_x(\varphi), & \text{if the firm exports.} \end{cases}$$

Model: Open Economy

Entry, Exit, and Export Status

- The export cost are equal across countries
  - a firm either exports in every period to all destinations or never export
- Export decision occurs after firms know their productivity φ: f_x is per period (per country) fixed costs of trade.
- Therefore,

$$\pi_d(\varphi) = \frac{r_d(\varphi)}{\sigma} - f$$
$$\pi_x(\varphi) = \frac{r_x(\varphi)}{\sigma} - f_x$$

- Total profits:

$$\pi(\varphi) = \pi_d(\varphi) + \max\left(0, n\pi_x(\varphi)\right).$$

Model: Open Economy

Entry, Exit, and Export Status

- By analogy with the closed economy, the firm value is given by

$$v(\varphi) = \max\left(0, \frac{\pi(\varphi)}{\delta}\right)$$

- The domestic cutoff:

$$\varphi^* = \inf\left(\varphi : v(\varphi) > 0\right)$$

- The exporting cutoff:

$$\varphi_x^* = \inf\left(\varphi : \varphi \geq \varphi^* \text{ and } \pi_x(\varphi) > 0\right)$$

- If for instance $\varphi^* = \varphi_x^*$, then all firms in the industry export, then the cutoff φ^* solves

$$\pi(\varphi^*) = \pi_d(\varphi^*) + n\pi_x(\varphi^*) = 0$$

- If $\varphi_x^* > \varphi^*$, then some firms produce only for their domestic market. Those firms do not export because of negative exporting profits. In this case,

$$\begin{aligned} \pi_d(\varphi^*) &= 0 \\ \pi_x(\varphi_x^*) &= 0. \end{aligned}$$

- Notice that it cannot be the case $\varphi_x^* < \varphi^*$!!!

- $\varphi_x^* > \varphi^*$ if and only if $\tau^{\sigma-1} f_x > f$ (this is our assumption).
- The distribution of productivities $\mu(\varphi)$ is the same as before. Namely,

$$\mu(\varphi) = \begin{cases} \frac{g(\varphi)}{1-G(\varphi^*)}, & \text{if } \varphi \geq \varphi^* \\ 0, & \text{otherwise.} \end{cases}$$

- The probability of successful entry is

$$p_{in} = 1 - G(\varphi^*).$$

- The probability of exporting (conditional on successful entry):

$$p_x = \frac{1 - G(\varphi_x^*)}{1 - G(\varphi^*)}.$$

- Let M denote the equilibrium mass of firms, then

$$M_x = p_x M$$

is the mass of exporting firms.

- Therefore,

$$M_t = M + nM_x$$

represents the total mass of varieties in any country.

Model: Open Economy

Aggregation

- It can be shown (see the class note) that

$$P = M_t^{\frac{1}{1-\sigma}} p(\tilde{\varphi}_t) = M_t^{\frac{1}{1-\sigma}} \frac{1}{\rho \tilde{\varphi}_t},$$

  where

$$\tilde{\varphi}_t = \left\{ \frac{1}{M_t} \left[M \left(\tilde{\varphi}(\varphi^*) \right)^{\sigma-1} + n M_x \left(\tau^{-1} \tilde{\varphi}(\varphi_x^*) \right)^{\sigma-1} \right] \right\}^{\frac{1}{\sigma-1}}$$

  is the weighted average productivity of all firms competing in a single market.

- $\tilde{\varphi}_t$ is the analogue of $\tilde{\varphi}$ in the closed economy.

- Furthermore (see the class note),

$$R = M_t r_d \left(\tilde{\varphi}_t \right)$$

- Finally, average revenues are given by

$$\bar{r} = \frac{R}{M} = r_d(\tilde{\varphi}(\varphi^*)) + np_x r_x(\tilde{\varphi}(\varphi_x^*))$$

- While average profits

$$\bar{\pi} = \pi_d(\tilde{\varphi}(\varphi^*)) + np_x \pi_x(\tilde{\varphi}(\varphi_x^*))$$

- The zero profit conditions imply that

$$\pi_d(\varphi^*) = 0 \iff \pi_d(\tilde{\varphi}(\varphi^*)) = f\left(\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right)$$

$$\pi_x(\varphi_x^*) = 0 \iff \pi_x(\tilde{\varphi}(\varphi_x^*)) = f_x\left(\left(\frac{\tilde{\varphi}(\varphi_x^*)}{\varphi_x^*}\right)^{\sigma-1} - 1\right)$$

- This implies that

$$\bar{\pi} = f\left(\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right) + np_x f_x\left(\left(\frac{\tilde{\varphi}(\varphi_x^*)}{\varphi_x^*}\right)^{\sigma-1} - 1\right)$$

Model: Open Economy

Equilibrium Conditions

- Finally,

$$\frac{r_x(\varphi_x^*)}{r_d(\varphi^*)} = \frac{f_x}{f} = \tau^{1-\sigma}\left(\frac{\varphi_x^*}{\varphi^*}\right)^{\sigma-1}.$$

- That is,

$$\varphi_x^* = \tau\varphi^*\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}.$$

- The free entry condition remains unchanged

$$\bar{\pi} = \frac{\delta f_e}{p_{in}}$$

- In addition,

$$\bar{\pi} = f\left(\left(\frac{\tilde{\varphi}(\varphi^*)}{\varphi^*}\right)^{\sigma-1} - 1\right) + np_x f_x \left(\left(\frac{\tilde{\varphi}(\varphi_x^*)}{\varphi_x^*}\right)^{\sigma-1} - 1\right)$$

$$\varphi_x^* = \tau\varphi^*\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}.$$

- The can solve for φ^* and $\bar{\pi}$.

- If we know φ^*, we can find φ_x^*, $\tilde{\varphi}_t$, $\tilde{\varphi}(\varphi^*)$, $\tilde{\varphi}(\varphi_x^*)$, p_{in}, and p_x.
- Along the steady state:

$$p_{in} M_e = \delta M$$

- This means that

$$R = L$$

- Finally, the mass of firms

$$M = \frac{R}{\bar{r}} = \frac{L}{\sigma\left(\bar{\pi} + f + p_x n f_x\right)}.$$

- Let us denote φ_a^* as the cutoff in autarky, then from the picture in the class, it can be seen that

$$\varphi^* > \varphi_a^*$$

- The least productive firms with $\varphi \in (\varphi_a^*, \varphi^*)$ no longer earn positive profits and, thereby, exit. It is consistent with the literature.
  - the reallocation of resources towards more productive firms
- Notice also that $\bar{\pi} > \bar{\pi}_a$, which implies that $M < M_a$ and the number of domestic producers will fall. However, as long as τ is not too high,

$$M_t = (1 + np_x)M > M_a.$$

Intuition:

- The elasticity of demand is unaffected by trade opening, so the fall in profit for domestic producers is not explained by a fall in mark-ups driven by increased foreign competition.
- The actual channel operates through the domestic factor market. In particular, trade translates into increased profitable opportunities for firms (recall that $\bar{\pi}$ goes up). This translates into more entry, thereby increasing labor demand and (given the fixed supply of labor) leading to a rise in the real wage $\frac{w}{P}$. This, in turn, brings down the profit level of the least productive firms to a level that forces them to exit.

- Welfare in autarky is given by

$$W_a = M_a^{\frac{1}{\sigma-1}} \rho \tilde{\varphi}\left(\varphi_a^*\right) = \rho \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \varphi_a^*$$

  Similarly,

$$W = \rho \left(\frac{L}{\sigma f}\right)^{\frac{1}{\sigma-1}} \varphi^*$$

- Since $\varphi^* > \varphi_a^*$, $W > W_a$. There are gains from trade.

- Finally, we are interested in showing that the model can replicate the type of market shares reallocations found in the data. In particular, it is possible to show

$$r_d(\varphi) < r_a(\varphi) < r_d(\varphi) + nr_x(\varphi) \quad \text{for any } \varphi \geq \varphi^*$$

- The first part of the inequality implies that all firms incur a loss in domestic sales in the open economy.
- The second part shows that an exporting firm has higher total revenues: a decrease in domestic sales is compensated by a rise in exports.
- Therefore, a firm who exports increase its share of industry revenues while a firm who does not export loses market share: redistribution (consistent with the data).