Spatial Competition with Heterogeneous Firms

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Motivation

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- So research questions from Trade naturally yield input for research in (applied) microeconomics.
- Example: Estimating impact on domestic profits of a tariff increase (Trade question). Standard econometric approach assumes unchanging product characteristics even after tariff increase. This, since no model available how product characteristics change if environment changes (Industrial Organization question). \(\Rightarrow\) Underestimation of effect of trade policy on market outcomes!
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- Example: Estimating impact on domestic profits of a tariff increase (Trade question). Standard econometric approach assumes unchanging product characteristics even after tariff increase. This, since no model available how product characteristics change if environment changes (Industrial Organization question). ⇒ Underestimation of effect of trade policy on market outcomes!
- Vogel 2008, JPE Vol. 116, no. 3 pp. 423-465 solves a stylized spatial competition model with endogenous product characteristics choice to confront this issue.
Model Set Up 1

Market is represented by a circle with unit circumference. Locations are indexed by $z \in [0, 1)$ and consumers (of Mass $L$) are uniformly distributed along circle. Each consumer buys either one unit of output from one of the firms ($N$ set of firms) or no unit at all (in that case utility is zero). In case of buying from firm $i$ he derives utility

$$u(z, i) = v - p_i - t \times D(z, i)$$

with $v$ common valuation of output, $p_i$ price of firm $i$ and $D(z, i)$ shortest path in circle from $z$ to $i$. $t > 0$ is the marginal "shopping" cost. Solution to consumer’s problem is

$$i \in \arg\min_{j \in N} \{ p_j + tD(z, j) \} \quad \text{and} \quad p_i + tD(z, i) \leq v$$

Tiebreak rule: if indifferent between two firms, buy from nearest one. We assume $v$ is so large that every costumer always buys one good.
Model Set Up 2

There are \( n \geq 2 \) firms. Firm \( i \)'s costs of supplying a consumer at \( z \) are

\[
k_i + 2\tau D(z, i)
\]

with \( k_i \) marginal costs of production, \( \bar{k} \) the average of marginal costs and \( 2\tau \) "shipping" costs with \( \tau \in [0, t) \). The game consists of two stages.

- **First Stage** Location stage: firms simultaneously choose their locations \( z_i \in [0, 1) \) with \( z \equiv (z_0, \ldots, z_{n-1}) \)

- **Second Stage** Price Stage: firms observe \( z \) and choose prices \( p_i \).

Strategy: Choice of probab. distribution over \([0, 1)\) and for each given \( z \in [0, 1) \) choice of a probab. distribution over prices.

Solution concept SPNE: each location is optimal given continuation game and each price distribution chosen in each continuation game is optimally chosen given strategies of other firms.
Market shares with no undercutting (i.e. there exists some indifferent consumer between all pairs of adjacent firms in the price stage, i.e. firm has positive market share):

\[ x^n_i = x_{i-1} + x_{i+1} = \frac{1}{2t} [p_{i-1} + p_{i+1} - 2p_i + t(d_{i-1,i} + d_{i,i+1})] \] (3)

with (for ex.) \( d_{i,i+1} \) the distance between adjacent firms \( i \) and \( i + 1 \).
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General market shares:

\[
x_i = \frac{1}{2t}[p_j + p_{j'} - 2p_i + t(D(i,j) + D(i,j'))]
\]

if \(j\) and \(j'\) are closest neighbors with some indifferent consumer in between. If \(\exists j\) s.t. \(p_i > p_j + tD(i,j)\) then of course \(x_i = 0\).

Also \(x_i = 1\) if \(\forall j \neq i\) \(p_i < p_j + tD(i,j)\). Note: \(x_i = x_i^n\) if no undercutting occurs.
Equilibrium Analysis 1

There is no pure strategy SPNE of this simple game!

**Reason**: Profits are not globally quasiconcave in own price, nor continuous.

![Figure 1: Market shares are discontinuous in prices.](image)

**Figure 1**: Market shares are discontinuous in prices.
Equilibrium Analysis 2

Approach in Paper to find mixed SPNE:

- Define auxiliary game: same as original game, only that now market shares are given by (3) and not (4). Profit function becomes continuous and quasiconcave in own price.

Figure 2: Market shares in auxiliary and real game.

- Auxiliary game has pure SPNE!
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Equilibrium Analysis 3

Proposition 1.

For any set of parameters $\theta \equiv (n, t, \tau, L)$ and $k \geq 0$ there exists a $\phi(\theta, k) > 0$ such that if $k_i \in [k, k + \phi(\theta, k)]$ for all $i$, then the set of SPNE-s is nonempty. Moreover the following properties hold for a subset $O^*$ of the SPNE-s:

1. Strategies are pure along the equilibrium path for every equilibrium in $O^*$.
2. For any order of firms around the circle there exists a corresponding SPNE in $O^*$.
3. Equilibrium in $O^*$ is characterized by

   \[
   d^*_{i, i+1} = \frac{1}{n} + \frac{2}{3t + 2\tau} \left( \bar{k} - \frac{k_i + k_{i+1}}{2} \right),
   \]

   \[
   p^*_i = (t + \tau) \left( \frac{1}{n} + \frac{2}{3t + 2\tau} \bar{k} \right) + \frac{t}{3t + 2\tau} k_i,
   \]

   \[
   x^*_i = \frac{1}{n} + \frac{2}{3t + 2\tau}(\bar{k} - k_i),
   \]

   \[
   \pi^*_i = Lt(x^*_i)^2.
   \]
Equilibrium Analysis 4

Proof Idea: Let $\pi_i^{A*}, \pi_i^*$ be respectively firm $i$-s profit in the auxiliary and real game along equilibrium path and $\pi_i^{A'}, E[\pi_i']$ firm $i$-s highest payoff from unilateral deviation from equilibrium path in auxiliary and real game. Author shows that if marginal costs of firms are similar enough then:

- $\pi_i^{A*} = \pi_i^*$ for all $i$
- $\pi_i^{A*} \geq \pi_i^{A'}$ for all $i$ if $\tau \geq 0$ and $\pi_i^{A*} > \pi_i^{A'}$ for all $i$ if $\tau > 0$.
- Either $\pi_i^{A'} \geq E[\pi_i']$ for all $i$ or $\pi_i^* > E[\pi_i']$ for all $i$

This implies that in any case $\pi_i^* \geq E[\pi_i']$ which is the SPNE-condition in the real game! Regarding uniqueness:

**Proposition 2.**

*If $\tau > 0$ and the $k_i$-s are similar enough, then any SPNE of the real game is strict along equilibrium path if and only if it is part of the set $O^*$ of Proposition 1*

Uniqueness important for comparative statics!
Price and market share depend on costs of competitors only through $\bar{k}$: each firm locates at center of its market share to minimize shipping costs $\implies$ location-adjusted prices of indifferent consumers are equal across firms $\implies$ profits depend only on average marginal costs in the market and own costs.
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A firm’s profit increases in its isolation $d_{i-1,i} + d_{i,i+1}$. i.e. there are two effects from a fall in $k_i$, prices are lower since costs are lower (classic argument) and additionally now less productive firms position themselves away from the more productive ones \( \implies \) more productive firms are more isolated, hence have more market power, hence higher profits. This tends to limit the extent of price fall. Unproductive firms specialize in niches. Implication for tariff example?
Uniqueness result means uniqueness in outcomes, since in fact, even for $\tau > 0$ multiple equilibria exist if firms have asymmetric marginal costs.
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Recalling formula for $d_{i,i+1}^*$ two neighbors produce varieties that are separated by more than $\frac{1}{n}$ iff their average cost is less than market average. I.e. there is a negative relationship between average costs of two direct competitors and their location in space.
Extension : Horizontal and Vertical Differentiation

The model above can be applied to markets with:

- Homogeneous goods which are differentiated by geographic locations
- Heterogeneous goods of different varieties, each customer has its ideal variety $z$ on the circle, its position.

$\implies$ Horizontal Differentiation only!

One can model additional vertical differentiation of goods by specifying anew the utility function:

$$u(z, i) = v - 1 + q_i^\gamma - p_i - tD(z, i) \text{ with } \gamma \in [0, 1) \quad (6)$$

where $q_i$ is the quality of the good produced by firm $i$. At first game stage firms now choose both position and quality. Basic insights are similar to first model, except that now more productive firms (in either quality or costs) don’t necessarily offer lower prices.
Thank you for your attention!