Online Appendix for "Affirmative Action: One Size Does Not Fit All"

Kala Krishna
The Pennsylvania State University
and NBER

Alexander Tarasov
University of Munich

July 2014

Appendix A (For Online Publication)

This Appendix contains the proofs of the results in the body of the paper.

Proof of Lemma 1. Follows from performing comparative statics on equations

$$1 - H(a^*) = \alpha, \tag{1}$$

$$s(a^*) - T = c(e^*(a^*, \tilde{P})). \tag{2}$$

Totally differentiating them gives

$$\begin{aligned} -h(a^*)da^* &= d\alpha \\ \{s'(a^*) - c'(e^*(a^*, \tilde{P}))e_a^*(a^*, \tilde{P})\}da^* - c'(e^*(a^*, \tilde{P}))e_{\tilde{P}}^*(a^*, \tilde{P})d\tilde{P} &= dT, \end{aligned}$$

where $s'(a^*) - c'(e^*(a^*, \tilde{P}))e_a^*(a^*, \tilde{P}) > 0$ as $e_a^*(a^*, \tilde{P}) < 0$ and $e_{\tilde{P}}^*(a^*, \tilde{P}) > 0$. Thus,

$$\begin{bmatrix} -h(a^*) & 0 \\ s'(a^*) - c'(e^*(a^*, \tilde{P}))e_a^*(a^*, \tilde{P}) & -c'(e^*(a^*, \tilde{P}))e_{\tilde{P}}^*(a^*, \tilde{P}) \end{bmatrix} \begin{bmatrix} da^* \\ d\tilde{P} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} d\alpha \\ dT \end{bmatrix}$$

so that

$$\begin{bmatrix} \frac{da^*}{d\alpha} & \frac{da^*}{dT} \\ \frac{d\tilde{P}}{d\alpha} & \frac{d\tilde{P}}{dT} \end{bmatrix} = \begin{bmatrix} -\frac{1}{h(a^*)} & 0 \\ -\frac{s'(.) - c'(e^*(.))e_a^*(.)}{h(a^*)c'(e^*(.))e_{\tilde{P}}^*(.)} & -\frac{1}{c'(e^*(.))e_{\tilde{P}}^*(.)} \end{bmatrix}.$$

■

Proof of Proposition 1

Recall that the social welfare is given by

$$\begin{aligned} W &= \gamma_1 \int_{a_N + a_A \geq a_1^*} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_1)) - F \right) dH_N(a_N) dH_A^1(a_A) \\ &\quad + \gamma_2 \int_{a_N + a_A \geq a_2^*} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_2)) - F \right) dH_N(a_N) dH_A^2(a_A). \end{aligned}$$

This can be rewritten in the following way (by adding and subtracting T for each agent):

$$\begin{aligned} W &= \gamma_1 \int_0^{a_{\max,1}^A} \left(\int_{a_1^* - a_A}^{a_{\max}^N} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_1)) - T \right) dH_N(a_N) \right) dH_A^1(a_A) \\ &\quad + \gamma_2 \int_0^{a_{\max,2}^A} \left(\int_{a_2^* - a_A}^{a_{\max}^N} \left(s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}_2)) - T \right) dH_N(a_N) \right) dH_A^2(a_A) \\ &\quad + \alpha(T - F). \end{aligned}$$

Then, adding and subtracting $s(a_N + a_A)$ to each of the above terms under the integral gives

$$\begin{aligned} W &= \gamma_1 \int_0^{a_{\max,1}^A} \left(\int_{a_1^* - a_A}^{a_{\max}^N} \begin{pmatrix} s(a_N + \beta a_A) - s(a_N + a_A) \\ + s(a_N + a_A) - c(e^*(a_N + a_A, \tilde{P}_1)) - T \end{pmatrix} dH_N(a_N) \right) dH_A^1(a_A) \\ &\quad + \gamma_2 \int_0^{a_{\max,2}^A} \left(\int_{a_2^* - a_A}^{a_{\max}^N} \begin{pmatrix} s(a_N + \beta a_A) - s(a_N + a_A) \\ + s(a_N + a_A) - c(e^*(a_N + a_A, \tilde{P}_2)) - T \end{pmatrix} dH_N(a_N) \right) dH_A^2(a_A) \\ &\quad + \alpha(T - F). \end{aligned}$$

Noting that

$$\begin{aligned} & \frac{\partial}{\partial a_i^*} \int_{a_i^* - a_A}^{a_{\max}^N} \left(s(a_N + \beta a_A) - s(a_N + a_A) + s(a_N + a_A) - c(e^*(a_N + a_A, \tilde{P}_i)) - T \right) dH_N(a_N) \\ = & -\left(s(a_i^* - (1-\beta) a_A) - s(a_i^*) + s(a_i^*) - c(e^*(a_i^*, \tilde{P}_i)) - T \right) h_N(a_i^* - a_A) \end{aligned}$$

and that at the cutoff for total ability, the marginal agent is indifferent between paying tuition and getting in and not, so that $c(e^*(a_i^*, \tilde{P}_i)) = s(a_i^*) - T$, it follows that

$$\begin{aligned} \frac{\partial W}{\partial \theta} &= \Bigg\{ -\gamma_1 \frac{\partial \tilde{P}_1}{\partial \theta} \int_0^{a_{\max,1}^A} \left(\int_{a_1^* - a_A}^{a_{\max}^N} c'(e^*(a_N + a_A, \tilde{P}_1)) \frac{\partial e^*(a_N + a_A, \tilde{P}_1)}{\partial \tilde{P}} dH_N(a_N) \right) dH_A^1(a_A) \\ &\quad - \gamma_2 \frac{\partial \tilde{P}_2}{\partial \theta} \int_0^{a_{\max,2}^A} \left(\int_{a_2^* - a_A}^{a_{\max}^N} c'(e^*(a_N + a_A, \tilde{P}_2)) \frac{\partial e^*(a_N + a_A, \tilde{P}_2)}{\partial \tilde{P}} dH_N(a_N) \right) dH_A^2(a_A) \Bigg\} \\ &\quad \Bigg\{ + \gamma_1 \frac{\partial a_1^*}{\partial \theta} \int_0^{a_{\max,1}^A} \left(s(a_1^*) - s(a_1^* - (1-\beta) a_A) \right) h_N(a_1^* - a_A) dH_A^1(a_A) \\ &\quad + \gamma_2 \frac{\partial a_2^*}{\partial \theta} \int_0^{a_{\max,2}^A} \left(s(a_2^*) - s(a_2^* - (1-\beta) a_A) \right) h_N(a_2^* - a_A) dH_A^2(a_A) \Bigg\}. \end{aligned}$$

Hence,

$$\frac{\partial W}{\partial \theta} = EE + SE,$$

where EE, the effort effect, is the first term in curly brackets and SE, the selection effect is the second.

Let us define θ^* as the non-discrimination quota. Under the non-discrimination quota, $\tilde{P}_1 = \tilde{P}_2 \equiv \tilde{P}$ and $a_1^* = a_2^* \equiv a^*$. Moreover, if $H_A^1(a_A) \equiv H_A^2(a_A)$, then $H^1(a) \equiv H^2(a)$. Then, from the equilibrium conditions given by

$$\gamma_i \left(1 - H^i(a_i^*)\right) = \theta_i \alpha, \tag{3}$$

$$s(a_i^*) - T = c(e^*(a_i^*, \tilde{P}_i)), \text{ where } i = 1, 2, \tag{4}$$

we obtain that

$$\frac{\gamma_1}{\gamma_2} = \frac{\theta_1}{\theta_2} \Leftrightarrow \theta_i = \gamma_i.$$

That is, the non-discrimination quota is equal to the share of the group in the total mass of agents. This makes sense as if the two groups are identical then their non discriminating quota share will be the same as their population weight.

Next, we assume that $H_A^1(a_A) \equiv H_A^2(a_A)$ and evaluate $\frac{\partial W}{\partial \theta}$ at the non-discrimination quota. From (3), it is straightforward to show that

$$\frac{\partial a_1^*}{\partial \theta}_{\theta=\theta^*} = \frac{\alpha}{\gamma_1 h^1(a^*)} > 0 \text{ and} \tag{5}$$

$$\frac{\partial a_2^*}{\partial \theta}_{\theta=\theta^*} = -\frac{\alpha}{\gamma_2 h^2(a^*)} < 0. \tag{6}$$

Hence, if $H^1(a) \equiv H^2(a)$,

$$\gamma_1 \frac{\partial a_1^*}{\partial \theta}_{\theta=\theta^*} = -\gamma_2 \frac{\partial a_2^*}{\partial \theta}_{\theta=\theta^*}.$$

This implies that SE evaluated at $\theta = \theta^*$ is equal to zero. From (4), it is also possible to show that

$$\frac{\partial \tilde{P}_1}{\partial \theta}_{\theta=\theta^*} = \frac{s'(a^*) - c'(e^*(a^*, \tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*, \tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}} \frac{\partial a_1^*}{\partial \theta}_{\theta=\theta^*} > 0, \text{ and} \tag{7}$$

$$\frac{\partial \tilde{P}_2}{\partial \theta}_{\theta=\theta^*} = \frac{s'(a^*) - c'(e^*(a^*, \tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*, \tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}} \frac{\partial a_2^*}{\partial \theta}_{\theta=\theta^*} < 0. \tag{8}$$

This then implies that

$$\gamma_1 \frac{\partial \tilde{P}_1}{\partial \theta}_{\theta=\theta^*} = -\gamma_2 \frac{\partial \tilde{P}_2}{\partial \theta}_{\theta=\theta^*}.$$

As a result, EE is also equal to zero. To summarize, if $H_A^1(a_A) \equiv H_A^2(a_A)$, then for all β,

$$\frac{\partial W(\theta)}{\partial \theta}_{\theta=\theta^*} = 0.$$

Thus, if W is well-behaved, i.e., it has a single peak as a function of θ, social welfare is maximized at $\theta = \theta^*$. However, this could also be a minimum, not a maximum as explored in the simulations.

The Proof of Proposition 2

Using the results derived in Proposition 2, the effort effect evaluated at the non-discrimination quota θ^* is given by

$$\begin{aligned} EE_{\theta=\theta^*} &= -\gamma_1 \frac{\partial \tilde{P}_1}{\partial \theta} \int_0^{a^A_{\max,1}} \left(\int_{a^*-a_A}^{a^N_{\max}} c'(e^*(a_N + a_A, \tilde{P})) \frac{\partial e^*(a_N + a_A, \tilde{P})}{\partial \tilde{P}} dH_N(a_N) \right) dH^1_A(a_A) \\ &\quad -\gamma_2 \frac{\partial \tilde{P}_2}{\partial \theta} \int_0^{a^A_{\max,2}} \left(\int_{a^*-a_A}^{a^N_{\max}} c'(e^*(a_N + a_A, \tilde{P})) \frac{\partial e^*(a_N + a_A, \tilde{P})}{\partial \tilde{P}} dH_N(a_N) \right) dH^2_A(a_A), \end{aligned}$$

as when θ is non discriminatory, $a_1^* = a_2^* = a^*$ and $\tilde{P}_1 = \tilde{P}_2 = \tilde{P}$. Note that[1]

$$\begin{aligned} &\int_0^{a^A_{\max,i}} \left(\int_{a^*-a_A}^{a^N_{\max}} c'(e^*(a_N + a_A, \tilde{P})) \frac{\partial e^*(a_N + a_A, \tilde{P})}{\partial \tilde{P}} dH_N(a_N) \right) dH^i_A(a_A) \\ =& \int_{a_N + a_A \geq a^*} c'(e^*(a_N + a_A, \tilde{P})) \frac{\partial e^*(a_N + a_A, \tilde{P})}{\partial \tilde{P}} dH_N(a_N) dH^i_A(a_A) \\ =& \int_{a^*}^{a_{\max,i}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^i(a), \end{aligned}$$

where $H^i(a)$ is the distribution of the total ability (on $[0, a_{\max,i}]$) in group i.

Thus, the effort effect can be written as:

$$EE_{\theta=\theta^*} = -\gamma_1 \frac{\partial \tilde{P}_1}{\partial \theta} \int_{a^*}^{a_{\max,1}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^1(a) - \gamma_2 \frac{\partial \tilde{P}_2}{\partial \theta} \int_{a^*}^{a_{\max,2}} c'(e^*(a, \tilde{P})) \frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} dH^2(a).$$

[1] By definition,

$$\begin{aligned} 1 - H^i(a) &= \int_a^{a_{\max,i}} h^i(a) da \\ &= \int_a^{a_{\max,i}} \left[\int_0^{a^A_{\max,i}} h_N(a - a_A) h^i_A(a_A) da_A \right] da \\ &= \int_0^{a^A_{\max,i}} h^i_A(a_A) \left[\int_a^{a_{\max,i}} h_N(a - a_A) da \right] da_A \\ &= \int_0^{a^A_{\max,i}} h^i_A(a_A) \left[H_N(a_{\max,i} - a_A) - H_N(a - a_A) \right] da_A. \end{aligned}$$

As $a_{\max,i} - a_A \geq a^N_{\max}$ for any a_A, $H_N(a_{\max,i} - a_A) = 1$. Hence,

$$\begin{aligned} \int_a^{a_{\max,i}} h^i(a) da &= \int_0^{a^A_{\max,i}} h^i_A(a_A) \left[1 - H_N(a - a_A) \right] da_A \\ &= \int_0^{a^A_{\max,i}} h^i_A(a_A) \left[\int_{a-a_A}^{a^N_{\max}} h_N(a_N) da_N \right] da_A \\ &= \int_0^{a^A_{\max,i}} \int_{a-a_A}^{a^N_{\max}} h^i_A(a_A) h_N(a_N) da_N da_A. \end{aligned}$$

Thus, when we integrate the change in effort over all agents whose effort changes, there is more than one way to do so: as above or alternatively, we could integrate $h^i(a)$ over all a above a^*.

Substituting the expressions for $\frac{\partial \tilde{P}_i}{\partial \theta}$ (see (7), (8) and (5), (6)), we obtain that

$$\begin{aligned} EE_{\theta=\theta^*} &= -\left[\frac{s'(a^*) - c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}}\right]\{\gamma_1\left[\frac{\alpha}{\gamma_1 h^1(a^*)}\right]\int_{a^*}^{a_{\max,1}} c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}dH^1(a) \\ &\quad -\gamma_2\left[\frac{\alpha}{\gamma_2 h^2(a^*)}\right]\int_{a^*}^{a_{\max,2}} c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}dH^2(a)\} \end{aligned}$$

Thus,

$$EE_{\theta=\theta^*} = D\left(\frac{\int_{a^*}^{a_{\max,2}} c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}dH^2(a)}{h^2(a^*)} - \frac{\int_{a^*}^{a_{\max,1}} c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}dH^1(a)}{h^1(a^*)}\right) \quad (9)$$

$$= D\left(\frac{\int_{a^*}^{a_{\max,2}} g(a,\tilde{P})dH^2(a)}{h^2(a^*)} - \frac{\int_{a^*}^{a_{\max,1}} g(a,\tilde{P})dH^1(a)}{h^1(a^*)}\right), \quad (10)$$

where recall $g(a,\tilde{P}) = c'(e^*(a,\tilde{P}))\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}}$ is positive (as $\frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}} = \frac{1}{f_e(.)} > 0$) and

$$D = \alpha\frac{s'(a^*) - c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*,\tilde{P}))\frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}} > 0.$$

Recall that, as $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ and the distribution of the natural ability is log-concave, we know that $H^1(a) \succeq_{LR} H^2(a)$ so that

$$\frac{h^1(a)}{h^1(a^*)} > \frac{h^2(a)}{h^2(a^*)} \text{ for any } a, a^* : a > a^*.$$

Moreover, as $a_{\max,i} = a^N_{\max} + a^A_{\max,i}$ and $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$, $a_{\max,1} \geq a_{\max,2}$. Hence, we have that

$$\begin{aligned} \frac{\int_{a^*}^{a_{\max,1}} g(a,\tilde{P})dH^1(a)}{h^1(a^*)} &\geq \frac{\int_{a^*}^{a_{\max,2}} g(a,\tilde{P})dH^1(a)}{h^1(a^*)} \\ &= \int_{a^*}^{a_{\max,2}} g(a,\tilde{P})\frac{h^1(a)}{h^1(a^*)}da \\ &> \int_{a^*}^{a_{\max,2}} g(a,\tilde{P})\frac{h^2(a)}{h^2(a^*)}da \\ &= \frac{\int_{a^*}^{a_{\max,2}} g(a,\tilde{P})dH^2(a)}{h^2(a^*)}. \end{aligned}$$

This implies that $EE_{\theta=\theta^*} < 0$.

The Proof of Proposition 3

From previous sections, the selection effect evaluated at the non-discrimination quota (which ensures $a_1^* = a_2^* = a^*$) is given by

$$\begin{aligned} SE_{\theta=\theta^*} &= \gamma_1\frac{\partial a_1^*}{\partial \theta}\int_0^{a^A_{\max,1}} (s(a^*) - s(a^* - (1-\beta)\,a_A))\,h_N(a^* - a_A)dH_A^1(a_A) \\ &\quad +\gamma_2\frac{\partial a_2^*}{\partial \theta}\int_0^{a^A_{\max,,2}} (s(a^*) - s(a^* - (1-\beta)\,a_A))\,h_N(a^* - a_A)dH_A^2(a_A). \end{aligned}$$

Substituting the expressions for $\frac{\partial a_i^*}{\partial \theta}$ (see (5) and (6)), we obtain that

$$\begin{aligned} SE_{\theta=\theta^*} &= \frac{\alpha}{h^1(a^*)}\int_0^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) h_N(a^* - a_A) dH^1_A(a_A) \\ &\quad -\frac{\alpha}{h^2(a^*)}\int_0^{a^A_{\max,2}} \left(s(a^*) - s(a^*_2 - (1-\beta)\, a_A)\right) h_N(a^* - a_A) dH^2_A(a_A). \end{aligned} \tag{11}$$

Taking into account that

$$h^i(a^*) = \int_0^{a^A_{\max,i}} h_N(a^* - y) h^i_A(y) dy, \tag{12}$$

the selection effect can be written as follows:

$$\begin{aligned} SE_{\theta=\theta^*} &= \alpha\int_0^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) \frac{h^1_A(a_A) h_N(a^* - a_A)}{\int_0^{a^A_{\max,1}} h_N(a^* - y) h^1_A(y) dy} da_A \\ &\quad -\alpha\int_0^{a^A_{\max,2}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) \frac{h^2_A(a_A) h_N(a^* - a_A)}{\int_0^{a^A_{\max,2}} h_N(a^* - y) h^2_A(y) dy} da_A. \end{aligned}$$

Next, we define

$$\tilde{h}^i_A(a_A, a^*) \equiv \frac{h^i_A(a_A) h_N(a^* - a_A)}{\int_0^{a^A_{\max,i}} h_N(a^* - y) h^i_A(y) dy}.$$

Suppressing a^* in the notation, we replace $\tilde{h}^i_A(a_A, a^*)$ with $\tilde{h}^i_A(a_A)$. Notice that $\tilde{h}^i_A(a_A)$ is a density function. Let $\tilde{H}^i_A(a_A)$ be its associated distribution function. As $H^1_A(a_A) \succeq_{LR} H^2_A(a_A)$,

$$\frac{\tilde{h}^1_A(a_A)}{\tilde{h}^1_A(x)} = \frac{h^1_A(a_A) h_N(a^* - a_A)}{h^1_A(x) h_N(a^* - x)} \geq \frac{h^2_A(a_A) h_N(a^* - a_A)}{h^2_A(x) h_N(a^* - x)} = \frac{\tilde{h}^2_A(a_A)}{\tilde{h}^2_A(x)} \text{ for any } a_A,\, x:\; a_A \geq x.$$

That is, $\tilde{H}^1_A(a_A) \succeq_{LR} \tilde{H}^2_A(a_A)$ implying $\tilde{H}^1_A(a_A) \succeq_1 \tilde{H}^2_A(a_A)$.

Then, the selection effect can be rewritten in the following way:

$$\begin{aligned} SE_{\theta=\theta^*} &= \alpha\int_0^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}^1_A(a_A) \\ &\quad -\alpha\int_0^{a^A_{\max,2}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}^2_A(a_A). \end{aligned}$$

Equivalently, as $H^1_A(a_A) \succeq_{LR} H^2_A(a_A)$ implies $a^A_{\max,1} \geq a^A_{\max,2}$

$$\begin{aligned} SE_{\theta=\theta^*} &= \alpha\int_{a^A_{\max,2}}^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}^1_A(a_A) \\ &\quad +\alpha\int_0^{a^A_{\max,2}} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\left(\tilde{H}^1_A(a_A) - \tilde{H}^2_A(a_A)\right). \end{aligned}$$

Integrating the second term above by parts implies that

$$\begin{aligned} SE_{\theta=\theta^*} &= \alpha \int_{a_{\max,2}^A}^{a_{\max,1}^A} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad -\alpha \left(s(a^*) - s(a^* - (1-\beta)\, a_{\max,2}^A)\right) \left(1 - \tilde{H}_A^1(a_{\max,2}^A)\right) \\ &\quad +\alpha\,(1-\beta) \int_0^{a_{\max,2}^A} \left(\tilde{H}_A^2(a_A) - \tilde{H}_A^1(a_A)\right) s'(a^* - (1-\beta)\, a_A) da_A \\ &> 0 \end{aligned}$$

Why? The third term is positive as $s'(\cdot)$ is positive and $\tilde{H}_A^2(a_A) \geq \tilde{H}_A^1(a_A)$ for any a_A (recall that $\tilde{H}_A^1(a_A) \succeq_1 \tilde{H}_A^2(a_A)$).

The sum of the first two terms is also positive as shown next. As $s(\cdot)$ is increasing, we know that

$$k(a_A) \equiv s(a^*) - s(a^* - (1-\beta)\, a_A) > 0$$

and $k(a_A)$ is increasing in a_A. Thus, the average area under the curve $k(a_A)$,

$$\int_{a_{\max,2}^A}^{a_{\max,1}^A} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) \; \frac{\tilde{h}_A^1(a_A)}{\left(1 - \tilde{H}_A^1(a_{\max,2}^A)\right)} da_A > s(a^*) - s(a^* - (1-\beta)\, a_{\max,2}^A)$$

which is its value at the lowest point. Thus, we have that

$$\alpha \int_{a_{\max,2}^A}^{a_{\max,1}^A} \left(s(a^*) - s(a^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \geq \alpha \left(s(a^*) - s(a^* - (1-\beta)\, a_{\max,2}^A)\right) (1 - \tilde{H}_A^1(a_{\max,2}^A)).$$

Thus, it is straightforward to see that $SE_{\theta=\theta^*} > 0$. Notice that if $\beta = 1$, then $SE_{\theta=\theta^*} = 0$.

The Proof of Proposition 4 (Example)

From (9), the effort effect evaluated at the non-discrimination quota is given by

$$EE_{\theta=\theta^*} = D \left(\frac{\int_{a^*}^{a_{\max,2}} c'(e^*(a,\tilde{P})) \frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}} dH^2(a)}{h^2(a^*)} - \frac{\int_{a^*}^{a_{\max,1}} c'(e^*(a,\tilde{P})) \frac{\partial e^*(a,\tilde{P})}{\partial \tilde{P}} dH^1(a)}{h^1(a^*)} \right),$$

where

$$D = \alpha \frac{s'(a^*) - c'(e^*(a^*,\tilde{P})) \frac{\partial e^*(a^*,\tilde{P})}{\partial a}}{c'(e^*(a^*,\tilde{P})) \frac{\partial e^*(a^*,\tilde{P})}{\partial \tilde{P}}}.$$

Given the assumptions, the effort effect can be written as follows (recall that agents with total ability higher than $\tilde{P}$ put in zero effort and, therefore, the upper bound of the integrals is $\min(\tilde{P}, a_{\max,i})$) :

$$EE_{\theta=\theta^*} = \alpha\,(S+C) \left(\frac{\int_{a^*}^{\min(\tilde{P},a_{\max,2})} h^2(a)da}{h^2(a^*)} - \frac{\int_{a^*}^{\min(\tilde{P},a_{\max,1})} h^1(a)da}{h^1(a^*)} \right).$$

Recall that

$$H^i(a) = \int_0^{a_{\max,i}^A} H_N(a-y) dH_A^i(y),$$

implying that

$$h^i(a) = \int_0^{a^A_{\max,i}} h_N(a-y) h^i_A(y) dy.$$

Since $H_N(\cdot)$ and $H^i_A(\cdot)$ are uniform,

$$h^i(a) = \frac{1}{a^A_{\max,i} a^N_{\max}} \int_{\max(0,a-a^N_{\max})}^{\min(a^A_{\max,i},a)} dy = \frac{\min(a^A_{\max,i},a) - \max(0,a-a^N_{\max})}{a^A_{\max,i} a^N_{\max}}.$$

This implies that if $a \geq a^* \geq \max\left[a^A_{\max,1}, a^A_{\max,2}, a^N_{\max}\right]$ (as assumed), then

$$\begin{aligned} h^i(a) &= \frac{a^A_{\max,i} + a^N_{\max} - a}{a^A_{\max,i} a^N_{\max}} \\ &= \frac{a_{\max,i} - a}{a^A_{\max,i} a^N_{\max}}, \end{aligned}$$

where $a_{\max,i} = a^A_{\max,i} + a^N_{\max}$. Hence,

$$\int_{a^*}^{\min(\tilde{P},a_{\max,i})} h^i(a) da = \frac{(a_{\max,i} - a^*)^2 - \left(a_{\max,i} - \min(\tilde{P}, a_{\max,i})\right)^2}{2 a^A_{\max,i} a^N_{\max}}.$$

Substituting the latter in the expression for the effort effect, we derive that

$$\begin{aligned} EE_{\theta=\theta^*} &= -\frac{\alpha(S+C)}{2} \frac{(a_{\max,1} - a^*)^2 - \left(a_{\max,1} - \min(\tilde{P}, a_{\max,1})\right)^2}{a_{\max,1} - a^*} \\ &\quad + \frac{\alpha(S+C)}{2} \frac{(a_{\max,2} - a^*)^2 - \left(a_{\max,2} - \min(\tilde{P}, a_{\max,2})\right)^2}{a_{\max,2} - a^*} \\ &= -\frac{\alpha(S+C)}{2} \left(a^A_{\max,1} - a^A_{\max,2} + \frac{\left(a_{\max,2} - \min(\tilde{P}, a_{\max,2})\right)^2}{a_{\max,2} - a^*} - \frac{\left(a_{\max,1} - \min(\tilde{P}, a_{\max,1})\right)^2}{a_{\max,1} - a^*} \right). \end{aligned}$$

From (11), the selection effect is given by

$$\begin{aligned} SE_{\theta=\theta^*} &= \frac{\alpha}{h^1(a^*)} \int_0^{a^A_{\max,1}} \left(s(a^*) - s(a^* - (1-\beta) a_A)\right) h_N(a^* - a_A) dH^1_A(a_A) \\ &\quad - \frac{\alpha}{h^2(a^*)} \int_0^{a^A_{\max,2}} \left(s(a^*) - s(a^* - (1-\beta) a_A)\right) h_N(a^* - a_A) dH^2_A(a_A). \end{aligned}$$

Taking into account the assumptions about the functional forms, the latter can be written as follows:

$$SE_{\theta=\theta^*} = \alpha(1-\beta) S \left(\frac{\int_0^{a^A_{\max,1}} a_A h_N(a^* - a_A) dH^1_A(a_A)}{h^1(a^*)} - \frac{\int_0^{a^A_{\max,2}} a_A h_N(a^* - a_A) dH^2_A(a_A)}{h^2(a^*)} \right).$$

We have that (recall $a^* > a^A_{\max,i}$)

$$\begin{aligned} \frac{\int_0^{a^A_{\max,i}} a_A h_N(a^* - a_A) dH^i_A(a_A)}{h^i(a^*)} &= \frac{\int_{a^* - a^N_{\max}}^{a^A_{\max,i}} a_A da_A}{a^A_{\max,i} + a^N_{\max} - a^*} \\ &= \frac{a^A_{\max,i} + a^* - a^N_{\max}}{2}. \end{aligned}$$

Therefore,

$$\begin{aligned} SE_{\theta=\theta^*} &= \alpha(1-\beta)S\left(\frac{a^A_{\max,1}+a^*-a^N_{\max}}{2}-\frac{a^A_{\max,2}+a^*-a^N_{\max}}{2}\right) \\ &= \frac{\alpha(1-\beta)S\left(a^A_{\max,1}-a^A_{\max,2}\right)}{2}. \end{aligned}$$

When Effort Affects the Payoffs from Education

In this section, we modify the model so that the effort put in is not fully wasted. We assume that the private gains from education are given by $s(a) - T + q(a,e)$, where $q(a,e)$ represents additional payoffs from effort.[2] We assume that $q(a,e)$ is increasing in both a and e, concave in e ($q_{ee}(a,e) < 0$), and the cross derivative $q_{ea}(a,e)$ is positive (which means that more able agents gain more from putting in more effort).

The sequence of actions is the same as in the benchmark model. An agent decides whether to take the exam or not and how much effort to put in (if she takes the exam). Let $e^*(a,\tilde{P})$ be the effort required to get in. It is defined as the solution of

$$\tilde{P} = f(a,e).$$

As can be seen, it is decreasing in a.

Let $\hat{e}(a)$ be the effort chosen if admission was ensured, i.e., effort independent of any considerations of admission. It is the solution of

$$\max_e \{s(a) - T + q(a,e) - c(e)\},$$

which is defined by

$$q_e(a,\hat{e}) - c'(\hat{e}) = 0.$$

Also, $q_{ee}(a,e) - c''(e) < 0$ so that $q_e(a,e) - c'(e) < 0$ for $e > \hat{e}(a)$. Also, as $q_{ea}(a,e)$ is positive, $\hat{e}(a)$ is increasing in a. Since $e^*(a,\tilde{P})$ is decreasing in a, there exists a unique ability where the two are equal. This cutoff ability is implicitly defined by

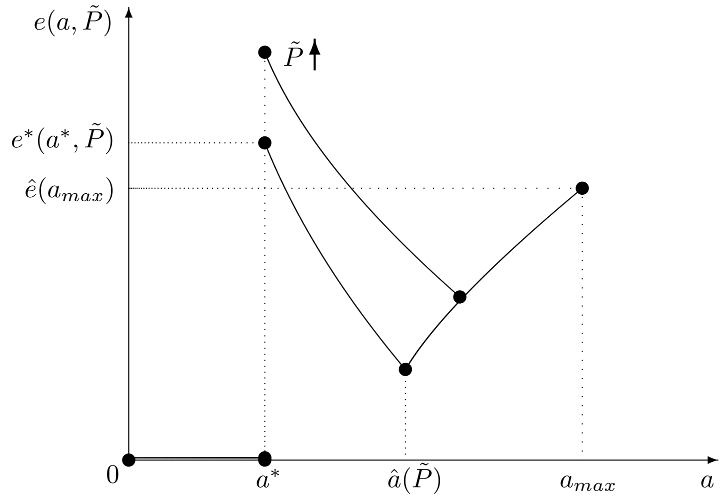
$$e^*(a,\tilde{P}) = \hat{e}(a)$$

and denoted by $\hat{a}(\tilde{P})$.

The equilibrium effort function is a composite one made up of $e^*(a,\tilde{P})$ and $\hat{e}(a)$. Agents with ability above $\hat{a}(\tilde{P})$ want to put in more effort than they need to to get in and so choose to put in what they want to independent of admission considerations. Agents with ability below $\hat{a}(\tilde{P})$ want to put in less effort

[2] This setup is equivalent to that where the function $s(\cdot)$ depends not only on a, but also on e.

Figure 1: Effort in the Model with Additional Payoffs from Effort



than they need to to get in and are forced to put in what is needed to be admitted. Hence, the agent with total ability a expends effort

$$e(a,\tilde{P}) = \max\{e^*(a,\tilde{P}),\hat{e}(a)\} \tag{13}$$

$$= \begin{cases} e^*(a,\tilde{P}) & \text{if } a \leq \hat{a}(\tilde{P}) \\ \hat{e}(a) & \text{if } a > \hat{a}(\tilde{P}). \end{cases} \tag{14}$$

As depicted in Figure 1, $e(a,\tilde{P})$ is decreasing in a till $\hat{a}(\tilde{P})$ and then increasing. A higher cutoff performance shifts the decreasing part of the curve upwards and to the right and does not effect the increasing part.

What about surplus? The surplus of an agent with ability a who decides to take the exam is given by

$$V(a,\tilde{P}) = s(a) - T + q(a,e(a,\tilde{P})) - c(e(a,\tilde{P})).$$

Taking into account the expression for $e(a,\tilde{P})$,

$$V(a,\tilde{P}) = \begin{cases} s(a) - T + q(a,e^*(a,\tilde{P})) - c(e^*(a,\tilde{P})) & \text{if } a \leq \hat{a}(\tilde{P}) \\ s(a) - T + q(a,\hat{e}(a)) - c(\hat{e}(a)) & \text{if } a > \hat{a}(\tilde{P}). \end{cases}$$

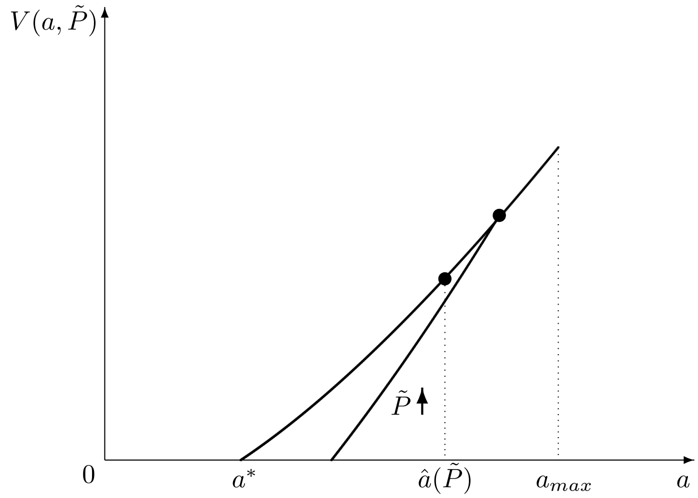
It is straightforward to show that if $a > \hat{a}(\tilde{P})$, $V(a,\tilde{P})$ is increasing in a. Indeed, by the envelope theorem, for $a > \hat{a}(\tilde{P})$

$$V_a(a,\tilde{P}) = s'(a) + q_a(a,\hat{e}(a)) > 0.$$

Next we show that $V(a,\tilde{P})$ is increasing in a for $a \leq \hat{a}(\tilde{P})$ as well. For $a \leq \hat{a}(\tilde{P})$,

$$V_a(a,\tilde{P}) = s'(a) + q_a(a,e^*(a,\tilde{P})) + (q_e(.) - c'(.))\, e^*_a(a,\tilde{P}).$$

Figure 2: The Surplus Function in the Model with Additional Payoffs from Effort



Note that for $a \leq \hat{a}(\tilde{P})$, $e(a,\tilde{P})) = e^*(a,\tilde{P}) > \hat{e}(a)$. That is, in this region effort is excessive so that $(q_e(.) - c'(.)) < 0$. Since $e^*_a(a,\tilde{P}) < 0$, $(q_e(.) - c'(.))\, e^*_a(a,\tilde{P}) > 0$. As a result, it follows that for $a \leq \hat{a}(\tilde{P})$, $\hat{V}_a(a,\tilde{P}) > 0$. Thus, we have shown that $V(a,\tilde{P})$ is increasing in a. Notice that a rise in $\tilde{P}$ raises the effort needed to get in and shifts $V(a,\tilde{P})$ downwards (for $a \leq \hat{a}(\tilde{P})$) and $\hat{a}(\tilde{P})$ up. This is depicted in Figure 2.

Finally, an agent with total ability a takes the exam if and only if her surplus from doing so is positive. Since this surplus is increasing in a, all agents with ability more that a some level take the exam. The cutoff ability, a^*, satisfies

$$V(a^*,\tilde{P}) = s(a^*) - T + q(a^*, e(a^*,\tilde{P})) - c(e(a^*,\tilde{P})) = 0, \tag{15}$$

as the outside option has been set at 0.

Hence, in the model when effort can be useful, there is still some wasted effort (when $a^* < \hat{a}(\tilde{P})$ in equilibrium). This occurs among the lower ability agents taking the exam. As a result, the effort distortion will again suggest that one discriminates in favor of the advantaged as they put in less wasteful effort. Hence, our results regarding the effort effect on welfare derived in the benchmark model can be derived in this modification of the model as well.

Appendix B (For Online Publication)

In this Appendix, we consider the extension of the benchmark model with two universities of different quality.

The Model

We assume that the universities are different in that they offer education of different qualities, which affects the payoffs from being educated. As a result, in equilibrium, the performance cutoff for a better university is higher so that it takes more effort to be accepted to the higher quality university. The payoffs from being educated at university i are given by $q^i s(a)$, where q^i is the measure of quality of university i (as before, a is the total ability). Here, $i \in \{H, L\}$. The net payoffs are given by

$$q^i s(a) - T_i - c(e^*(a, \tilde{P}^i)),$$

where T_i is the tuition fee and $\tilde{P}^i$ is the performance cutoff at university i ($\tilde{P}^H$ is assumed to be higher than $\tilde{P}^L$ (see the discussion below)), $e^*(a, \tilde{P}^i)$ is the effort level put in to be accepted. $c(.)$ is weakly convex.

Lemma 1 below shows that the difference between the net payoffs from studying in the better university is increasing in ability. As a result, more able agents are matched with better universities.

Lemma 1 *For any given performance cutoffs,*

$$\begin{aligned} D(a; \tilde{P}^H, \tilde{P}^L) &= q^H s(a) - T_H - c(e^*(a, \tilde{P}^H)) - \left(q^L s(a) - T_L - c(e^*(a, \tilde{P}^L))\right) \\ &= \triangle q s(a) - \triangle T - c(e^*(a, \tilde{P}^H)) + c(e^*(a, \tilde{P}^L)) \end{aligned}$$

where $\triangle q = q^H - q^L > 0$ *and* $\triangle T = T_H - T_L$. *Then,*

$$\begin{aligned} \frac{\partial D(a; \tilde{P}^H, \tilde{P}^L)}{\partial a} &= \triangle q s'(a) + \left[-c'(e^*(a, \tilde{P}^H))\frac{\partial e^*(a, \tilde{P}^H)}{\partial a} - \left(-c'(e^*(a, \tilde{P}^L))\frac{\partial e^*(a, \tilde{P}^L)}{\partial a}\right)\right] && (16) \\ &= \triangle q s'(a) - \left[c'(e^*(a, \tilde{P}^H)) - c'(e^*(a, \tilde{P}^L))\right] \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} && (17) \\ &\quad + c'(e^*(a, \tilde{P}^L))\left(\frac{\partial e^*(a, \tilde{P}^L)}{\partial a} - \frac{\partial e^*(a, \tilde{P}^H)}{\partial a}\right) && (18) \\ &> 0. && (19) \end{aligned}$$

Proof. Using the fact that

$$f(a, e^*(a, \tilde{P})) = \tilde{P},$$

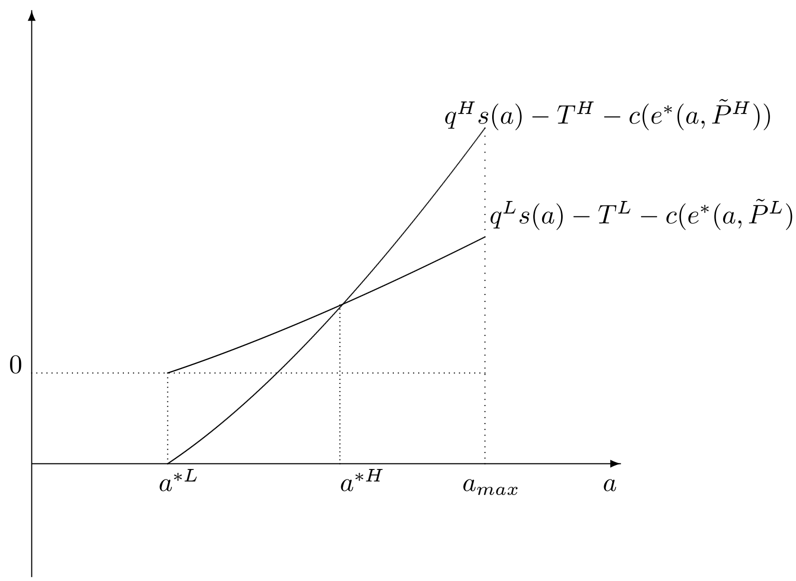
it is easy to see that

$$e^*_{\tilde{P}}(a, \tilde{P}) = \frac{1}{f_e(a, e^*(a, \tilde{P}))} > 0.$$

That is, meeting a higher cutoff requires greater effort from any agent. Thus, as $c(\cdot)$ is convex, $c'(e^*(a, \tilde{P}^H)) > c'(e^*(a, \tilde{P}^L))$. In addition,

$$e^*_a(a, \tilde{P}) = -\frac{f_a(a, e^*(a, \tilde{P}))}{f_e(a, e^*(a, \tilde{P}))} < 0. \quad (20)$$

Figure 3: Payoffs from Education: Two Schools



This implies that

$$-\left[c'(e^*(a, \tilde{P}^H)) - c'(e^*(a, \tilde{P}^L))\right] \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} > 0.$$

Finally,

$$e^*_{a\tilde{P}}(a, \tilde{P}) = -\frac{f_{ea}(a, e^*(a, \tilde{P})) + f_{ee}(a, e^*(a, \tilde{P}))e^*_a(a, \tilde{P})}{\left(f_e(a, e^*(a, \tilde{P}))\right)^2} < 0,$$

as $f_{ee} < 0$, $e^*_a(a, \tilde{P}) < 0$, and $f_{ea} > 0$. This in turn means that $e^*_a(a, \tilde{P}^L) > e^*_a(a, \tilde{P}^H)$, implying that

$$c'(e^*(a, \tilde{P}^L)) \left(\frac{\partial e^*(a, \tilde{P}^L)}{\partial a} - \frac{\partial e^*(a, \tilde{P}^H)}{\partial a} \right) > 0.$$

Summarizing the above findings, it follows that

$$\frac{\partial D(a; \tilde{P}^H, \tilde{P}^L)}{\partial a} > 0.$$

■

It is probably easiest to see what we have in a picture like Figure 3. We have that $q^H s(a)$ is steeper than $q^L s(a)$, as $q^H > q^L$ and $s(a)$ is increasing in ability so that more able individuals earn more at any given education level, and this is more so at better institutions. In order to get in to school H (or L), each agent must put in $e^*(a, \tilde{P}^H)$ (or $e^*(a, \tilde{P}^L)$) and this means costs of $c(e^*(a, \tilde{P}^H))$ (and $c(e^*(a, \tilde{P}^L))$) be incurred. These costs are decreasing in ability as the more able need to put in less effort to meet any given performance cutoff. Moreover, they decrease faster in ability when the cutoff is higher (as shown in Lemma 1). This happens because the higher performance cutoff requires more effort from all individuals, but due to the complementarity between ability and effort in creating performance, more able agents need to put in less effort to attain the higher cutoff. As they are putting in less effort to get the lower performance cutoff anyway, this increased effort to meet a higher cutoff is also less costly for them.

Thus, the net surplus (the benefit less the cost) from going to school is increasing in ability, and more so for the better school as depicted in Figure 3. This means that when we add tuition cost which are independent of ability, the net benefit of going to the better school rises faster than that of the worse school so that these curves can cross at most once and better students must select into the better school. Note this is independent of tuition, though too high a tuition could make the payoff from that school lie entirely below that of the other so no one goes there.

Next we consider the equilibrium and comparative statics of the model. In the equilibrium, there are two total ability cutoffs: a_H^* and a_L^*. Agents with ability lower than a_L^* choose the outside option. The cutoffs are determined by taking the number of seats in the better school and finding a_H^* such that these seats are filled. a_L^* is then defined by its seats being filled by lower ability agents. This gives the equilibrium conditions:

$$1 - H(a_H^*) = \alpha_H, \tag{21}$$

$$H(a_H^*) - H(a_L^*) = \alpha_L, \tag{22}$$

where α_i is the number of seats in university i and $H(.)$ is the distribution of total ability. As before, we assume that the natural and acquired abilities are independently distributed across the agents. The distribution functions are given by $H_N(a_N)$ and $H_A(a_A)$ on $[0, a_{\max}^N]$ and $[0, a_{\max}^A]$, respectively. Then, the distribution function for the total ability a is $H(a)$ on $[0, a_{\max}]$, where

$$H(a) = \int_0^{a_{\max}^A} H_N(a-y)dH_A(y) \tag{23}$$

and $a_{\max} = a_{\max}^N + a_{\max}^A$.

The agent at a_L^* must be indifferent between the worse school and the outside option of zero which pins down $c(e^*(a_L^*, \tilde{P}^L))$ and defines $\tilde{P}^L$. The agent at a_H^* must be indifferent between the two schools which pins down $c(e^*(a_H^*, \tilde{P}^H))$ and defines $\tilde{P}^H$. Thus

$$q^L s(a_L^*) - T_L - c(e^*(a_L^*, \tilde{P}^L)) = 0, \tag{24}$$

$$\triangle q s(a_H^*) - \triangle T - c(e^*(a_H^*, \tilde{P}^H)) + c(e^*(a_H^*, \tilde{P}^L)) = 0. \tag{25}$$

Thus, we have four unknowns: a_H^*, a_L^*, $\tilde{P}^H$, $\tilde{P}^L$; and four equilibrium equations. Note that the condition, $\tilde{P}^H > \tilde{P}^L$, is equivalent to $c(e^*(a_H^*, \tilde{P}^H)) - c(e^*(a_H^*, \tilde{P}^L)) > 0$. Therefore, from the equilibrium conditions, we can infer that $\tilde{P}^H > \tilde{P}^L$ if and only if $\triangle q s(a_H^*) - \triangle T > 0$. In other words, that the difference in the tuition levels is not set too high relative to the difference in quality.

Next, we explore how changes in T_i affect the equilibrium outcome. The following lemma holds.

Lemma 2 *1) The cutoffs, a_H^* and a_L^*, do not depend on the tuition fees, T_H and T_L.*

2) A rise in T_H does not affect $\tilde{P}^L$ and decreases $\tilde{P}^H$.

3) A rise in T_L decreases $\tilde{P}^L$ and increases $\tilde{P}^H$.

4) A rise in T_H and T_L (such that $\triangle T$ does not change) decreases $\tilde{P}^L$ and $\tilde{P}^H$.

Proof. 1)-2) and 4) directly follow from the equilibrium equations. Let us prove the third statement in the lemma. From the equilibrium, we have that

$$\frac{\partial \tilde{P}^L}{\partial T_L} = -\frac{1}{c'\left(e^*(a_L^*, \tilde{P}^L)\right)\frac{\partial e^*(a_L^*, \tilde{P}^L)}{\partial \tilde{P}}} < 0.$$

In addition,

$$\frac{\partial \tilde{P}^H}{\partial T_L} = -\frac{-c'\left(e^*(a_H^*, \tilde{P}^L)\right)\frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}}\frac{\partial \tilde{P}^L}{\partial T_L} - 1}{c'\left(e^*(a_H^*, \tilde{P}^H)\right)\frac{\partial e^*(a_H^*, \tilde{P}^H)}{\partial \tilde{P}}} = \frac{c'\left(e^*(a_H^*, \tilde{P}^L)\right)\frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}}\frac{\partial \tilde{P}^L}{\partial T_L} + 1}{c'\left(e^*(a_H^*, \tilde{P}^H)\right)\frac{\partial e^*(a_H^*, \tilde{P}^H)}{\partial \tilde{P}}}.$$

Hence, the sign of the derivative is the same as the sign of the numerator (as the denominator is positive). The numerator is in turn equal to

$$c'\left(e^*(a_H^*, \tilde{P}^L)\right)\frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}}\frac{\partial \tilde{P}^L}{\partial T_L} + 1 = 1 - \frac{c'\left(e^*(a_H^*, \tilde{P}^L)\right)\frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}}}{c'\left(e^*(a_L^*, \tilde{P}^L)\right)\frac{\partial e^*(a_L^*, \tilde{P}^L)}{\partial \tilde{P}}}.$$

Note that as $a_L^* < a_H^*$, $e^*(a_H^*, \tilde{P}^L) < e^*(a_L^*, \tilde{P}^L)$ implying that $c'\left(e^*(a_H^*, \tilde{P}^L)\right) < c'\left(e^*(a_L^*, \tilde{P}^L)\right)$. Moreover, since $\frac{\partial e^*(a, \tilde{P})}{\partial \tilde{P}} = 1/f_e\left(a, e^*(a, \tilde{P})\right)$,

$$\frac{\frac{\partial e^*(a_H^*, \tilde{P}^L)}{\partial \tilde{P}}}{\frac{\partial e^*(a_L^*, \tilde{P}^L)}{\partial \tilde{P}}} = \frac{f_e\left(a_L^*, e^*(a_L^*, \tilde{P}^L)\right)}{f_e\left(a_H^*, e^*(a_H^*, \tilde{P}^L)\right)} < 1,$$

as $f_{ea} > 0$ and $f_{ee} < 0$. That is, the sign of the numerator is positive. This proves the statement. ■

The intuition behind 1) and 2) in the lemma is straightforward. The idea behind 3) is as follows. Keep the performance cutoffs fixed. An increase in T_L shifts the payoff curve for L down and reduces a_H^* while raising a_L^*. As a result, more agents apply for the seats in the high-quality university and fewer for the low quality one. As the number of seats remains the same, $\tilde{P}^H$ must rise and $\tilde{P}^L$ fall.

The intuition behind 4 is simple. Suppose we increase tuition fees by the same amount. At given performance cutoffs, this change does not affect the intersection of the two curves as both sift down by the same amount, but raises a_L^*. This reduces the demand for school L below its capacity which reduces the performance cutoff of L. This fall in L's performance cutoff must shift the payoff for L back up so that it goes through the original level of a_L^*. However, the fall in L's performance cutoff has a smaller impact on the payoff for higher ability agents and so makes it flatter. This reduces a_H^* from its original level, requiring a fall in H's performance cutoff as well.[3]

Next, we explore the effects of changes in the number of available seats on the equilibrium outcomes. Using the equilibrium equations we see that a rise in α_L does not change a_H^* (as it is pinned down by α_H) and decreases a_L^*. This in turn means that $\tilde{P}^L$ and $\tilde{P}^H$ fall. Intuitively, more available seats in the low-quality university reduces the performance cutoff in that university, making it more attractable compared to the high-quality university. As the number of seats in the high-quality university does not change, the performance cutoff, $\tilde{P}^H$, must fall to compensate for the decrease in $\tilde{P}^L$.

A rise in α_H in turn decreases both the ability cutoffs, a_L^* and a_H^*. The decrease in a_L^* in turn results in lower $\tilde{P}^L$. The low-quality university has to reduce its performance cutoff in order to fill in the all available seats. The impact on $\tilde{P}^H$ is also straightforward. The direct effect of a rise in α_H decreases a_H^*, reducing $\tilde{P}^H$. In addition, the rise in α_H decreases $\tilde{P}^L$, which further reduces $\tilde{P}^H$ (see (24)). As can be seen, both effects work in the same direction. As a result, $\tilde{P}^H$ falls. The following lemma summarizes the above reasoning.

Lemma 3 *1) A rise in α_L does not change a_H^* and decreases a_L^*, $\tilde{P}^L$, and $\tilde{P}^H$.*

2) A rise in α_H decreases a_H^* and a_L^* and $\tilde{P}^L$ and $\tilde{P}^H$.

Next, we examine the welfare implications of changes in the parameters in the model.

Social Welfare

As before, we allow the private gains from education to differ from the social gains. Specifically, for an individual, natural and acquired abilities are of the same importance but for the society natural ability is more important than acquired ability.

Social welfare is given by (the outside option is normalized to zero)

$$\begin{aligned} W \;=\; & \int_{a_N+a_A\geq a_H^*} \Big(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^H)) - F\Big)\, dH_N(a_N)dH_A(a_A) \\ & + \int_{a_L^*\leq a_N+a_A<a_H^*} \Big(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^L)) - F\Big)\, dH_N(a_N)dH_A(a_A). \end{aligned} \tag{26}$$

where F is the social cost of education per student. Note that as the tuition is a lump-sum transfer, T does not directly affect the welfare. It only affects it via the effort put in by agents.

Next, we explore the effects of the tuition fees on the social welfare. First, we examine how changes in T_L and T_H affect the welfare. Then, we find the values of T_L and T_H that maximize the social welfare

[3] Formally, this result comes from the impact of T_H on $\tilde{P}^H$ being stronger than that of T_L.

function. It is straightforward to see that

$$\begin{aligned}\frac{\partial W}{\partial T_i} &= \int_{a_N+a_A\geq a_H^*} \frac{\partial\left(q^H s(a_N+\beta a_A)-c(e^*(a_N+a_A,\tilde{P}^H))-F\right)}{\partial T_i} dH_N(a_N)dH_A(a_A)\\ &+\int_{a_L^*\leq a_N+a_A<a_H^*} \frac{\partial\left(q^L s(a_N+\beta a_A)-c(e^*(a_N+a_A,\tilde{P}^L))-F\right)}{\partial T_i} dH_N(a_N)dH_A(a_A).\end{aligned}$$

Here, we take into account that the ability cutoffs do not depend on the tuition fees (see Lemma 1)).

From the results stated in Lemma 1, we can conclude that (recall that $\frac{\partial \tilde{P}^L}{\partial T_H}=0$)

$$\frac{\partial W}{\partial T_H} = -\frac{\partial \tilde{P}^H}{\partial T_H}\int_{a_N+a_A\geq a_H^*} c'(e^*(a_N+a_A,\tilde{P}^H))\frac{\partial e^*(a_N+a_A,\tilde{P}^H)}{\partial \tilde{P}} dH_N(a_N)dH_A(a_A) > 0,$$

while

$$\begin{aligned}\frac{\partial W}{\partial T_L} &= -\frac{\partial \tilde{P}^H}{\partial T_L}\int_{a_N+a_A\geq a_H^*} c'(e^*(a_N+a_A,\tilde{P}^H))\frac{\partial e^*(a_N+a_A,\tilde{P}^H)}{\partial \tilde{P}} dH_N(a_N)dH_A(a_A)\\ &-\frac{\partial \tilde{P}^L}{\partial T_L}\int_{a_L^*\leq a_N+a_A<a_H^*} c'(e^*(a_N+a_A,\tilde{P}^H))\frac{\partial e^*(a_N+a_A,\tilde{P}^L)}{\partial \tilde{P}} dH_N(a_N)dH_A(a_A).\end{aligned}$$

The sign of the latter expression is ambiguous, as $\frac{\partial \tilde{P}^H}{\partial T_L} > 0$ and $\frac{\partial \tilde{P}^L}{\partial T_L} < 0$.

As can be seen, the impact of T_H on welfare is similar to that in the model with one university: $\frac{\partial W}{\partial T_H} > 0$. The intuition is similar as well. A rise in T_H reduces the effort put in the agents who decide to apply for the high-quality university and does not change the effort of the agents who apply for the low-quality university. As a result, welfare rises. The impact of T_L is ambiguous in general. A rise in T_L reduces the effort put in by the agents applying for the low-quality university and increases the effort put in by the agents applying for the high-quality university. As a result, given T_H, there exists a certain optimal level of T_L such that $\frac{\partial W}{\partial T_L} = 0$ (unless the condition $\frac{\partial W}{\partial T_L} = 0$ delivers the minimum).

However, if the goal is to describe the value of the pair (T_H, T_L) that delivers the maximum, the outcome will be exactly the same as in the case with one university. In other words, the social welfare as a function of T_H and T_L is maximized when the effort put in by the marginal agents is equal to zero. That is, $e^*(a_H^*, \tilde{P}^H) = 0$ and $e^*(a_L^*, \tilde{P}^L) = 0$. This can be obviously seen from the expression for the social welfare (26), which is maximized when there is no wasted effort. The conditions of having zero effort put in by the marginal agents can be written as follows:

$$s(a_H^*) = \frac{T_H - T_L}{\triangle q}, \tag{27}$$

$$s(a_L^*) = T_L/q^L. \tag{28}$$

Since the ability cutoffs are determined by the number of seats in the universities, from the above equations we can find the optimal values of the tuition levels.

Note that if we assume that $T_H = T_L = T$, then the welfare function will be increasing in T. However, the optimal value of T does not elicit the zero effort put in by all agents. Indeed, a rise in T reduces $\tilde{P}^L$ and, thereby, $\tilde{P}^H$ (recall that $\triangle T = 0$). In this case, it is straightforward to show that the social welfare is increasing in T. Therefore, we keep increasing T till the effort put in by the marginal agent a_L^* (a further increase in T does not affect welfare, as $\tilde{P}^L$ is not affected anymore). The equilibrium conditions in this case are

$$\begin{aligned} \triangle q s(a_H^*) - c(e^*(a_H^*, \tilde{P}^H)) + c(e^*(a_H^*, \tilde{P}^L)) &= 0, \\ q^L s(a_L^*) - T &= 0. \end{aligned}$$

As $c(e^*(a_L^*, \tilde{P}^L)$ is equal to zero, $c(e^*(a_H^*, \tilde{P}^L))$ is equal to zero as well. Therefore, the equilibrium conditions can be written as follows:

$$\begin{aligned} \triangle q s(a_H^*) - c(e^*(a_H^*, \tilde{P}^H)) &= 0, \\ q^L s(a_L^*) - T &= 0. \end{aligned}$$

As can be seen, $e^*(a_H^*, \tilde{P}^H)$ is strictly positive in the equilibrium. That is, the agents applying for the high-quality university put in some positive effort. The corresponding value of $\tilde{P}^H$ can be found from the first equation in the latter system of equations.

Finally, similar to the benchmark case with one university, the distortion caused by selection into education can not be completely removed, as the social gains from education are different from the private gains.

The Case with Quotas

In this section, we introduce educational quotas in the above framework. We assume there are two groups of agents indexed by $i \in \{1, 2\}$, which have identical distributions of natural ability and potentially different distributions of acquired ability. The latter is motivated by the fact that agents with different social backgrounds have had different educational inputs prior to taking the exam, which in turn results in different acquired abilities on their part. In particular, we assume that $H_N^1(a_N) = H_N^2(a_N) \equiv H_N(a_N)$, while $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ where $\succeq_{LR}$ stands for the likelihood stochastic order. Hence,

$$\frac{h_A^1(a_A)}{h_A^1(x)} > \frac{h_A^2(a_A)}{h_A^2(x)} \text{ for any } a_A, x : a_A > x.$$

This means that group 1 is more favored in terms of acquired ability than group 2. In addition, we assume that the distribution of natural ability has a log-concave density. This assumption is needed to ensure the likelihood stochastic order of the distributions of total ability: i.e., $H^1(a) \succeq_{LR} H^2(a)$.[4]

[4] See Theorem 1.C.9 in Shaked and Shanthikumar (2007) for the proof. This assumption is not very restrictive, as a number of commonly used distributions such as the normal, uniform, Gamma, and Beta distributions satisfy it.

The share of each group in the total mass of agents (which is normalized to unity) is denoted by γ_i, where $\gamma_1 + \gamma_2 = 1$. We then define θ_{ik} as a share of available seats reserved for group i in university k: $\theta_{1k} + \theta_{2k} = 1$ for $k \in \{H, L\}$. If these quotas are binding, then the cutoffs for the two groups will differ. Note that the quota given to a certain group can be in general different in different universities. The equilibrium conditions can be then written as follows:

$$\begin{aligned}
\gamma_i \left(1 - H^i(a^*_{iH})\right) &= \theta_{iH}\alpha_H, \\
\gamma_i \left(H^i(a^*_{iH}) - H^i(a^*_{iL})\right) &= \theta_{iL}\alpha_L, \\
q^L s(a^*_{iL}) - T_L - c(e^*(a^*_{iL}, \tilde{P}^{iL})) &= 0, \\
\triangle q s(a^*_{iH}) - \triangle T - c(e^*(a^*_{iH}, \tilde{P}^{iH})) + c(e^*(a^*_{iH}, \tilde{P}^{iL})) &= 0,
\end{aligned}$$

where $i \in \{1, 2\}$.

We define by a non-discrimination quota in university k, θ^*_k (the quota in favor of group 1, the corresponding quota in favor of group 2 is $1 - \theta^*_k$), such that the quota leads to $\tilde{P}^{1k} = \tilde{P}^{2k}$: i.e., the performance cutoffs are the same for both groups. If both universities set the non-discrimination quotas, then it is straightforward to see that

$$\begin{aligned}
a^*_{1H} &= a^*_{2H}, \\
a^*_{1L} &= a^*_{2L}.
\end{aligned}$$

If in addition $H^1(a) \equiv H^2(a) = H(a)$, then

$$\theta^*_H = \theta^*_L = \gamma_1.$$

This is similar to the case with one university.

Next, we write down the social welfare under the presence of two groups of agents. In particular, we have the following expression:

$$\begin{aligned}
W &= \sum_i \gamma_i \int_{a_N + a_A \geq a^*_{iH}} \left(q^H s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iH})) - F\right) dH_N(a_N) dH^i_A(a_A) \qquad (29) \\
&\quad + \sum_i \gamma_i \int_{a^*_{iL} \leq a_N + a_A < a^*_{iH}} \left(q^L s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iL})) - F\right) dH_N(a_N) dH^i_A(a_A) \\
&= \sum_i \gamma_i \int_{a_N + a_A \geq a^*_{iH}} \left(q^H s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iH})) - T_H\right) dH_N(a_N) dH^i_A(a_A) \\
&\quad + \sum_i \gamma_i \int_{a^*_{iL} \leq a_N + a_A < a^*_{iH}} \left(q^L s(a_N + \beta a_A) - c(e^*(a_N + a_A, \tilde{P}^{iL})) - T_L\right) dH_N(a_N) dH^i_A(a_A) \\
&\quad + \alpha_H(T_H - F) + \alpha_L(T_L - F).
\end{aligned}$$

In the next sections, we explore the behavior of social welfare (as a function of the quotas) around the non-discrimination quotas.

Symmetric Groups with no Selection Effect

In this subsection, we assume that the groups are symmetric: $H^1(a) \equiv H^2(a) = H(a)$; and examine how *uniform* changes in the quotas set by the universities locally affect the social welfare in the case of no selection effect. In particular, we assume that

$$\begin{aligned} \theta_{1k} &= \mu\theta_k^*, \text{ implying that} \\ \theta_{2k} &= 1 - \mu\theta_k^*. \end{aligned}$$

This specification allows us consider uniform changes in the quotas set by the universities. Moreover, if $\mu = 1$, then both universities set the non-discrimination quotas θ_k^*. If $\mu = 0$, then in both universities all seats are given to the second group. Finally, if $\mu = 1/\gamma_1 > 1$, then all seats in both universities are given to the first group (recall that if $H^1(a) \equiv H^2(a)$, $\theta_H^* = \theta_L^* = \gamma_1$). Next, we consider the social welfare as a function of μ in the case of no selection effect: $\beta = 1$.

Taking into account (29), the derivative of the social welfare function with respect to μ can be written as follows. Note that $W = W(\tilde{P}^{iH}, \tilde{P}^{iL}, a_{iH}^*, a_{iL}^*)$. Thus:

$$\begin{aligned} \frac{\partial W(.)}{\partial \mu} &= \frac{\partial W(.)}{\partial \tilde{P}^{iH}} \frac{\partial \tilde{P}^{iH}}{\partial \mu} + \frac{\partial W(.)}{\partial \tilde{P}^{iL}} \frac{\partial \tilde{P}^{iL}}{\partial \mu} \\ &+ \frac{\partial W(.)}{\partial a_{iH}^*} \frac{\partial a_{iH}^*}{\partial \mu} + \frac{\partial W(.)}{\partial a_{iL}^*} \frac{\partial \tilde{P}^{iL}}{\partial \mu}. \end{aligned}$$

$$\begin{aligned} \frac{\partial W(.)}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a_{iH}^*} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &- \sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a_{iL}^* \leq a < a_{iH}^*} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) \\ &- \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \left(q^H s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) - T_H \right) h^i(a_{iH}^*) \\ &+ \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \left(q^L s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iL})) - T_L \right) h^i(a_{iH}^*) \\ &- \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \left(q^L s(a_{iL}^*) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L \right) h^i(a_{iL}^*). \end{aligned}$$

Taking into account the equilibrium conditions for the marginal agents, these can be written as follows:

$$\begin{aligned}\frac{\partial W(.)}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &\quad - \sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) \\ &\quad - \sum_i \gamma_i \frac{\partial a^*_{iH}}{\partial \mu} \left(q^L s(a^*_{iH}) - c(e^*(a^*_{iH}, \tilde{P}^{iL})) - T_L \right) h^i(a^*_{iH}) \\ &\quad + \sum_i \gamma_i \frac{\partial a^*_{iH}}{\partial \mu} \left(q^L s(a^*_{iH}) - c(e^*(a^*_{iH}, \tilde{P}^{iL})) - T_L \right) h^i(a^*_{iH}) \\ &\quad - \sum_i \gamma_i \frac{\partial a^*_{iL}}{\partial \mu} (0)\, h^i(a^*_{iL}).\end{aligned}$$

(as agent a^*_{iH} is indifferent between schools) so that

$$\begin{aligned}\frac{\partial W}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &\quad - \sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a).\end{aligned}$$

Let us then find the derivative of $\tilde{P}^{ik}$ with respect to μ. From the equilibrium conditions, we have

$$\frac{\partial \tilde{P}^{iL}}{\partial \mu} = \frac{q_L s'(a^*_{iL}) - c'(e^*(a^*_{iL}, \tilde{P}^{iL})) \frac{\partial e^*(a^*_{iL}, \tilde{P}^{iL})}{\partial a}}{c'(e^*(a^*_{iL}, \tilde{P}^{iL})) \frac{\partial e^*(a^*_{iL}, \tilde{P}^{iL})}{\partial \tilde{P}}} \frac{\partial a^*_{iL}}{\partial \mu}.$$

Taking into account that

$$\theta_{iH}\alpha_H + \theta_{iL}\alpha_L = \gamma_i \left(1 - H^i(a^*_{iL})\right),$$

we derive that

$$\begin{aligned}\frac{\partial a^*_{1L}}{\partial \mu} &= -\frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{\gamma_1 h(a^*_{1L})} < 0, \\ \frac{\partial a^*_{2L}}{\partial \mu} &= \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{\gamma_2 h(a^*_{2L})} > 0.\end{aligned}$$

This implies that

$$\begin{aligned}\frac{\partial \tilde{P}^{1L}}{\partial \mu} &= -\frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L}, \tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L}, \tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L}, \tilde{P}^{1L})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{\gamma_1 h(a^*_{1L})} < 0, \\ \frac{\partial \tilde{P}^{2L}}{\partial \mu} &= \frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L}, \tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L}, \tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L}, \tilde{P}^{2L})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{\gamma_2 h(a^*_{2L})} > 0.\end{aligned}$$

As can be seen, at the non-discrimination quotas (when $\mu = 1$),

$$\gamma_1 \frac{\partial \tilde{P}^{1L}}{\partial \mu} = -\gamma_2 \frac{\partial \tilde{P}^{2L}}{\partial \mu},$$

implying that (recall that $H^1(a) \equiv H^2(a)$)

$$\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \le a < a^*_{iH}} c'(e^*(a, \tilde{P}^{iL})) \frac{\partial e^*(a, \tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) = 0.$$

Next, we consider the derivative of $\tilde{P}^{iH}$ with respect to μ. From the equilibrium conditions, we have that

$$\frac{\partial \tilde{P}^{iH}}{\partial \mu} = \frac{\left[\triangle q s'(a^*_{iH}) - c'(e^*(a^*_{iH}, \tilde{P}^{iH}))\frac{\partial e^*(a^*_{iH}, \tilde{P}^{iH})}{\partial a} + c'(e^*(a^*_{iH}, \tilde{P}^{iL}))\frac{\partial e^*(a^*_{iH}, \tilde{P}^{iL})}{\partial a}\right] \frac{\partial a^*_{iH}}{\partial \mu}}{c'(e^*(a^*_{iH}, \tilde{P}^{iH}))\frac{\partial e^*(a^*_{iH}, \tilde{P}^{iH})}{\partial \tilde{P}}}$$
$$+ \frac{c'(e^*(a^*_{iH}, \tilde{P}^{iL}))\frac{\partial e^*(a^*_{iH}, \tilde{P}^{iL})}{\partial \tilde{P}} \frac{\partial \tilde{P}^{iL}}{\partial \mu}}{c'(e^*(a^*_{iH}, \tilde{P}^{iH}))\frac{\partial e^*(a^*_{iH}, \tilde{P}^{iH})}{\partial \tilde{P}}}.$$

In addition, we have that

$$\frac{\partial a^*_{1H}}{\partial \mu} = -\frac{\theta^*_H \alpha_H}{\gamma_1 h(a^*_{1H})} < 0, \ \frac{\partial a^*_{2H}}{\partial \mu} = \frac{\theta^*_H \alpha_H}{\gamma_2 h(a^*_{2H})} > 0.$$

Summarizing all the previous results, we can see that

$$\frac{\partial \tilde{P}^{1H}}{\partial \mu} < 0 \text{ and } \frac{\partial \tilde{P}^{2H}}{\partial \mu} > 0.$$

The latter follows from the fact that $\frac{\partial \tilde{P}^{1L}}{\partial \mu} < 0$, $\frac{\partial \tilde{P}^{2L}}{\partial \mu} > 0$, and

$$\triangle q s'(a^*_{iH}) - c'(e^*(a^*_{iH}, \tilde{P}^{iH}))\frac{\partial e^*(a^*_{iH}, \tilde{P}^{iH})}{\partial a} + c'(e^*(a^*_{iH}, \tilde{P}^{iL}))\frac{\partial e^*(a^*_{iH}, \tilde{P}^{iL})}{\partial a} > 0.$$

Moreover, if $\mu = 1$, it is straightforward to see that

$$\gamma_1 \frac{\partial \tilde{P}^{1H}}{\partial \mu} = -\gamma_2 \frac{\partial \tilde{P}^{2H}}{\partial \mu},$$

implying that

$$\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \ge a^*_{iH}} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) = 0.$$

Thus, we have that

$$\frac{\partial W}{\partial \mu}|_{\mu=1} = 0.$$

That is, non-discrimination delivers a local extremum. In the case of concave welfare social welfare, non-discrimination is globally optimal. Next, we explore the case when the groups are asymmetric in terms of the distribution of total ability.

Intuitively the logic is exactly the same. When the two groups are the same, the losses of one group exactly make up for the gains of the other for slight changes. Thus, if welfare is concave, this is a local maximum.

Asymmetric Groups with no Selection Effect

Assume now that $H^1(a) \succeq_{LR} H^2(a)$. Using the results derived in the above section, the derivative of welfare with respect to μ is given by

$$\begin{aligned}\frac{\partial W}{\partial \mu} &= -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a,\tilde{P}^{iH})) \frac{\partial e^*(a,\tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\ &\quad -\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a,\tilde{P}^{iL})) \frac{\partial e^*(a,\tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a).\end{aligned}$$

Consider the second component of the derivative:

$$\begin{aligned}& -\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a,\tilde{P}^{iL})) \frac{\partial e^*(a,\tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) \\ = \;& (\theta^*_H \alpha_H + \theta^*_L \alpha_L) \left[\begin{array}{l} \frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L},\tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L},\tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial \tilde{P}}} \int_{a^*_{1L} \leq a < a^*_{1H}} c'(e^*(a,\tilde{P}^{1L})) \frac{\partial e^*(a,\tilde{P}^{1L})}{\partial \tilde{P}} \frac{h^1(a)}{h^1(a^*_{1L})} da \\ - \frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L},\tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L},\tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial \tilde{P}}} \int_{a^*_{2L} \leq a < a^*_{2H}} c'(e^*(a,\tilde{P}^{2L})) \frac{\partial e^*(a,\tilde{P}^{2L})}{\partial \tilde{P}} \frac{h^2(a)}{h^2(a^*_{2L})} da \end{array} \right].\end{aligned}$$

At the non-discrimination quota, $a^*_{2L} = a^*_{1L}$ and $\tilde{P}^{2L} = \tilde{P}^{1L}$. This implies that

$$\frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L},\tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L},\tilde{P}^{2L})) \frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial \tilde{P}}} = \frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L},\tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L},\tilde{P}^{1L})) \frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial \tilde{P}}}.$$

Moreover, as $H^1(a) \succeq_{LR} H^2(a)$,

$$\frac{h^1(a)}{h^1(a^*_{1L})} \geq \frac{h^2(a)}{h^2(a^*_{2L})} \text{ for any } a > a^*_{1L} = a^*_{2L}.$$

Thus,

$$\int_{a^*_{1L} \leq a < a^*_{1H}} c'(e^*(a,\tilde{P}^{1L})) \frac{\partial e^*(a,\tilde{P}^{1L})}{\partial \tilde{P}} \frac{h^1(a)}{h^1(a^*_{1L})} da > \int_{a^*_{2L} \leq a < a^*_{2H}} c'(e^*(a,\tilde{P}^{2L})) \frac{\partial e^*(a,\tilde{P}^{2L})}{\partial \tilde{P}} \frac{h^2(a)}{h^2(a^*_{2L})} da,$$

implying that

$$-\sum_i \gamma_i \frac{\partial \tilde{P}^{iL}}{\partial \mu} \int_{a^*_{iL} \leq a < a^*_{iH}} c'(e^*(a,\tilde{P}^{iL})) \frac{\partial e^*(a,\tilde{P}^{iL})}{\partial \tilde{P}} dH^i(a) > 0$$

when evaluated at the non-discrimination quota ($\mu = 1$).

Consider then the first component of the derivative, which is given by

$$-\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a,\tilde{P}^{iH})) \frac{\partial e^*(a,\tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a).$$

From the previous section, we have that

$$\begin{aligned}\frac{\partial \tilde{P}^{iH}}{\partial \mu} &= \frac{\left[\triangle q s'(a^*_{iH}) - c'(e^*(a^*_{iH},\tilde{P}^{iH})) \frac{\partial e^*(a^*_{iH},\tilde{P}^{iH})}{\partial a} + c'(e^*(a^*_{iH},\tilde{P}^{iL})) \frac{\partial e^*(a^*_{iH},\tilde{P}^{iL})}{\partial a}\right] \frac{\partial a^*_{iH}}{\partial \mu}}{c'(e^*(a^*_{iH},\tilde{P}^{iH})) \frac{\partial e^*(a^*_{iH},\tilde{P}^{iH})}{\partial \tilde{P}}} \\ &\quad + \frac{c'(e^*(a^*_{iH},\tilde{P}^{iL})) \frac{\partial e^*(a^*_{iH},\tilde{P}^{iL})}{\partial \tilde{P}} \frac{\partial \tilde{P}^{iL}}{\partial \mu}}{c'(e^*(a^*_{iH},\tilde{P}^{iH})) \frac{\partial e^*(a^*_{iH},\tilde{P}^{iH})}{\partial \tilde{P}}}.\end{aligned}$$

This means that at the non-discrimination quotas,

$$\begin{aligned}
\gamma_1 \frac{\partial \tilde{P}^{1H}}{\partial \mu} &= -\frac{\triangle q s'(a^*_{1H}) - c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{iH})}{\partial a} + c'(e^*(a^*_{1H}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1H})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H}{h^1(a^*_{1H})} \\
&\quad - \frac{c'(e^*(a^*_{1H}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1L})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L},\tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L},\tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1H})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{h^1(a^*_{1L})}, \\
\gamma_2 \frac{\partial \tilde{P}^{2H}}{\partial \mu} &= \frac{\triangle q s'(a^*_{2H}) - c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2H})}{\partial a} + c'(e^*(a^*_{2H}, \tilde{P}^{2L}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2H})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H}{h^2(a^*_{2H})} \\
&\quad + \frac{c'(e^*(a^*_{2H}, \tilde{P}^{iL}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{iL})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L},\tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L},\tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2H})}{\partial \tilde{P}}} \frac{\theta^*_H \alpha_H + \theta^*_L \alpha_L}{h^2(a^*_{2L})}.
\end{aligned}$$

Note that at the non-discrimination quota:

$$\begin{aligned}
& \theta^*_H \alpha_H \frac{\triangle q s'(a^*_{1H}) - c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{iH})}{\partial a} + c'(e^*(a^*_{1H}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1H})}{\partial \tilde{P}}} \\
= \; & \theta^*_H \alpha_H \frac{\triangle q s'(a^*_{2H}) - c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2H})}{\partial a} + c'(e^*(a^*_{2H}, \tilde{P}^{2L}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2H})}{\partial \tilde{P}}} \equiv A.
\end{aligned}$$

Moreover, at the non-disrimination quota:

$$\begin{aligned}
& (\theta^*_H \alpha_H + \theta^*_L \alpha_L) \frac{c'(e^*(a^*_{1H}, \tilde{P}^{1L}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1L})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{1L}) - c'(e^*(a^*_{1L},\tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial a}}{c'(e^*(a^*_{1L},\tilde{P}^{1L}))\frac{\partial e^*(a^*_{1L},\tilde{P}^{1L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{1H}, \tilde{P}^{1H}))\frac{\partial e^*(a^*_{1H},\tilde{P}^{1H})}{\partial \tilde{P}}} \\
= \; & (\theta^*_H \alpha_H + \theta^*_L \alpha_L) \frac{c'(e^*(a^*_{2H}, \tilde{P}^{iL}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{iL})}{\partial \tilde{P}} \left(\frac{q_L s'(a^*_{2L}) - c'(e^*(a^*_{2L},\tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial a}}{c'(e^*(a^*_{2L},\tilde{P}^{2L}))\frac{\partial e^*(a^*_{2L},\tilde{P}^{2L})}{\partial \tilde{P}}} \right)}{c'(e^*(a^*_{2H}, \tilde{P}^{2H}))\frac{\partial e^*(a^*_{2H},\tilde{P}^{2H})}{\partial \tilde{P}}} \equiv B.
\end{aligned}$$

Thus, at the non-disrimination quota:

$$\begin{aligned}
\gamma_1 \frac{\partial \tilde{P}^{1H}}{\partial \mu} &= -\frac{A}{h^1(a^*_H)} - \frac{B}{h^1(a^*_L)}, \\
\gamma_2 \frac{\partial \tilde{P}^{2H}}{\partial \mu} &= \frac{A}{h^2(a^*_H)} + \frac{B}{h^2(a^*_L)}.
\end{aligned}$$

As a result, at the non-disrimination quota:

$$\begin{aligned}
& -\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu} \int_{a \geq a^*_{iH}} c'(e^*(a, \tilde{P}^{iH})) \frac{\partial e^*(a, \tilde{P}^{iH})}{\partial \tilde{P}} dH^i(a) \\
= \; & \left(\frac{A}{h^1(a^*_H)} + \frac{B}{h^1(a^*_L)} \right) \int_{a \geq a^*_H} c'(e^*(a, \tilde{P}^H)) \frac{\partial e^*(a, \tilde{P}^H)}{\partial \tilde{P}} h^1(a) da \\
& - \left(\frac{A}{h^2(a^*_H)} + \frac{B}{h^2(a^*_L)} \right) \int_{a \geq a^*_H} c'(e^*(a, \tilde{P}^H)) \frac{\partial e^*(a, \tilde{P}^H)}{\partial \tilde{P}} h^2(a) da.
\end{aligned}$$

Taking into account the stochastic order of the distributions of total ability, it is straightforward to see that

$$
\begin{aligned}
A\int_{a\geq a_H^*} c'(e^*(a,\tilde{P}^H))\frac{\partial e^*(a,\tilde{P}^H)}{\partial \tilde{P}}\frac{h^1(a)}{h^1(a_H^*)}da &> A\int_{a\geq a_H^*} c'(e^*(a,\tilde{P}^H))\frac{\partial e^*(a,\tilde{P}^H)}{\partial \tilde{P}}\frac{h^2(a)}{h^2(a_H^*)}da,\\
B\int_{a\geq a_H^*} c'(e^*(a,\tilde{P}^H))\frac{\partial e^*(a,\tilde{P}^H)}{\partial \tilde{P}}\frac{h^1(a)}{h^1(a_L^*)}da &> B\int_{a\geq a_H^*} c'(e^*(a,\tilde{P}^H))\frac{\partial e^*(a,\tilde{P}^H)}{\partial \tilde{P}}\frac{h^2(a)}{h^2(a_L^*)}da.
\end{aligned}
$$

In other words, at the non-discrimination quota:

$$
-\sum_i \gamma_i \frac{\partial \tilde{P}^{iH}}{\partial \mu}\int_{a\geq a_{iH}^*} c'(e^*(a,\tilde{P}^{iH}))\frac{\partial e^*(a,\tilde{P}^{iH})}{\partial \tilde{P}}dH^i(a) > 0.
$$

To sum up, we have shown that the derivative of social welfare with respect to μ evaluated at the non-discrimination quota is positive. This means that discriminating in favor of group 1 locally increases the social welfare. This results is the same as that in the case of one university. That is, the effort effect works in favor of the advantaged group.

$$
EE_{\mu=1} > 0.
$$

This makes sense as weaker students need to put in more effort to get in and this effort is wasteful. So discriminating against the less advantaged group raises welfare. Next, we explore the role of the selection effect.

The Selection Effect

Recall that the social welfare when $\beta < 1$ is given by

$$
\begin{aligned}
W &= \sum_i \gamma_i \int_{a_N+a_A\geq a_{iH}^*}\left(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iH})) - F\right)dH_N(a_N)dH_A^i(a_A)\\
&\quad +\sum_i \gamma_i \int_{a_{iL}^*\leq a_N+a_A< a_{iH}^*}\left(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iL})) - F\right)dH_N(a_N)dH_A^i(a_A)\\
&= \sum_i \gamma_i \int_{a_N+a_A\geq a_{iH}^*}\left(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iH})) - T_H\right)dH_N(a_N)dH_A^i(a_A)\\
&\quad +\sum_i \gamma_i \int_{a_{iL}^*\leq a_N+a_A< a_{iH}^*}\left(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iL})) - T_L\right)dH_N(a_N)dH_A^i(a_A)\\
&\quad +\alpha_H(T_H-F)+\alpha_L(T_L-F).
\end{aligned}
$$

The latter can be written as follows:

$$
\begin{aligned}
W &= \sum_i \gamma_i \int_0^{a^A_{\max,i}}\left(\int_{a_{iH}^*-a_A}^{a^N_{\max}}\left(q^H s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iH})) - T_H\right)dH_N(a_N)\right)dH_A^i(a_A)\\
&\quad +\sum_i \gamma_i \int_0^{a^A_{\max,i}}\left(\int_{a_{iL}^*-a_A}^{a_{iH}^*-a_A}\left(q^L s(a_N+\beta a_A) - c(e^*(a_N+a_A,\tilde{P}^{iL})) - T_L\right)dH_N(a_N)\right)dH_A^i(a_A)\\
&\quad +\alpha_H(T_H-F)+\alpha_L(T_L-F).
\end{aligned}
$$

When we explore the selection effect only, by definition we look at the effect via the cutoffs directly and not via the performance cutoffs. Thus, the selection effect is as follows:

$$
\begin{aligned}
SE &= -\sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(q^H s(a_{iH}^* - a_A + \beta a_A) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) - T_H\right) h_N(a_{iH}^* - a_A) dH_A^i(a_A) \\
&\quad + \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(q^L s(a_{iH}^* - a_A + \beta a_A) - c(e^*(a_{iH}^*, \tilde{P}^{iL})) - T_L\right) h_N(a_{iH}^* - a_A) dH_A^i(a_A) \\
&\quad - \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(q^L s(a_{iL}^* - a_A + \beta a_A) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L\right) h_N(a_{iL}^* - a_A) dH_A^i(a_A) \\
&= -\sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(\begin{array}{c} \triangle q s(a_{iH}^* - a_A + \beta a_A) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) \\ + c(e^*(a_{iH}^*, \tilde{P}^{iL})) - \triangle T \end{array} \right) h_N(a_{iH}^* - a_A) dH_A^i(a_A) \\
&\quad - \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(q^L s(a_{iL}^* - a_A + \beta a_A) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L\right) h_N(a_{iL}^* - a_A) dH_A^i(a_A).
\end{aligned}
$$

Taking into account the equilibrium conditions, we know that

$$
\begin{aligned}
\triangle q s(a_{iH}^*) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) + c(e^*(a_{iH}^*, \tilde{P}^{iL})) - \triangle T &= 0 \\
q^L s(a_{iL}^*) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L &= 0
\end{aligned}
$$

so that have

$$
\begin{aligned}
& \triangle q s(a_{iH}^* - a_A + \beta a_A) - c(e^*(a_{iH}^*, \tilde{P}^{iH})) + c(e^*(a_{iH}^*, \tilde{P}^{iL})) - \triangle T \\
=\ & \triangle q \left(s(a_{iH}^* - a_A + \beta a_A) - s(a_{iH}^*)\right), \text{ and} \\
& q^L s(a_{iL}^* - a_A + \beta a_A) - c(e^*(a_{iL}^*, \tilde{P}^{iL})) - T_L \\
=\ & q^L \left(s(a_{iL}^* - a_A + \beta a_A) - s(a_{iL}^*)\right).
\end{aligned}
$$

Hence,

$$
\begin{aligned}
SE &= -\triangle q \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_{iH}^* - a_A + \beta a_A) - s(a_{iH}^*)\right) h_N(a_{iH}^* - a_A) dH_A^i(a_A) \qquad (30) \\
&\quad - q^L \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_{iL}^* - a_A + \beta a_A) - s(a_{iL}^*)\right) h_N(a_{iL}^* - a_A) dH_A^i(a_A).
\end{aligned}
$$

Note that in the case of one university we have only the second component of the above expression. However, we can apply the technique developed for the case with one university to both components, as they have similar functional forms.

Consider, for instance, the first term in the above expression given by:

$$
SE_1 = -\triangle q \sum_i \gamma_i \frac{\partial a_{iH}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_{iH}^* - a_A + \beta a_A) - s(a_{iH}^*)\right) h_N(a_{iH}^* - a_A) dH_A^i(a_A).
$$

Recall that

$$\frac{\partial a_{1H}^*}{\partial \mu} = -\frac{\theta_H^* \alpha_H}{\gamma_1 h(a_{1H}^*)} < 0, \;\; \frac{\partial a_{2H}^*}{\partial \mu} = \frac{\theta_H^* \alpha_H}{\gamma_2 h(a_{2H}^*)} > 0.$$

Hence, SE_1 (evaluated at the non-discrimination quota) can be written as follows:

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= \triangle q \alpha_H \theta_H^* \int_0^{a_{\max,1}^A} \left(s(a_H^* - a_A + \beta a_A) - s(a_H^*)\right) \frac{h_N(a_H^* - a_A) h_A^1(a_A)}{h(a_H^*)} da_A \\ &\quad - \triangle q \alpha_H \theta_H^* \int_0^{a_{\max,2}^A} \left(s(a_H^* - a_A + \beta a_A) - s(a_H^*)\right) \frac{h_N(a_H^* - a_A) h_A^2(a_A)}{h(a_H^*)} da_A. \end{aligned}$$

As in the benchmark case, we consider the following density functions:

$$\tilde{h}_A^i(a_A) \equiv \frac{h_A^i(a_A) h_N(a_H^* - a_A)}{\int_0^{a_{\max,i}^A} h_N(a_H^* - y) h_A^i(y) dy}.$$

Let $\tilde{H}_A^i(a_A)$ be its associated distribution function. As $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$,

$$\frac{\tilde{h}_A^1(a_A)}{\tilde{h}_A^1(x)} = \frac{h_A^1(a_A) h_N(a_H^* - a_A)}{h_A^1(x) h_N(a_H^* - x)} \geq \frac{h_A^2(a_A) h_N(a_H^* - a_A)}{h_A^2(x) h_N(a_H^* - x)} = \frac{\tilde{h}_A^2(a_A)}{\tilde{h}_A^2(x)} \text{ for any } a_A,\, x:\;\; a_A \geq x.$$

That is, $\tilde{H}_A^1(a_A) \succeq_{LR} \tilde{H}_A^2(a_A)$ implying $\tilde{H}_A^1(a_A) \succeq_1 \tilde{H}_A^2(a_A)$.

Then, SE_1 can be rewritten in the following way:

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= -\triangle q \alpha_H \theta_H^* \int_0^{a_{\max,1}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad + \triangle q \alpha_H \theta_H^* \int_0^{a_{\max,2}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^2(a_A). \end{aligned}$$

Equivalently, as $H_A^1(a_A) \succeq_{LR} H_A^2(a_A)$ implies $a_{\max,1}^A \geq a_{\max,2}^A$

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= -\triangle q \alpha_H \theta_H^* \int_{a_{\max,2}^A}^{a_{\max,1}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad - \triangle q \alpha_H \theta_H^* \int_0^{a_{\max,2}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\left(\tilde{H}_A^1(a_A) - \tilde{H}_A^2(a_A)\right). \end{aligned}$$

Integrating the second term above by parts implies that

$$\begin{aligned} SE_1|_{\theta=\theta^*} &= -\triangle q \alpha_H \theta_H^* \int_{a_{\max,2}^A}^{a_{\max,1}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) \\ &\quad + \triangle q \alpha_H \theta_H^* \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_{\max,2}^A)\right) \left(1 - \tilde{H}_A^1(a_{\max,2}^A)\right) \\ &\quad - \triangle q \alpha_H \theta_H^* (1-\beta) \int_0^{a_{\max,2}^A} \left(\tilde{H}_A^2(a_A) - \tilde{H}_A^1(a_A)\right) s'(a_H^* - (1-\beta)\, a_A) da_A \\ &< 0, \end{aligned}$$

as $s'(\cdot)$ is positive and $\tilde{H}_A^2(a_A) \geq \tilde{H}_A^1(a_A)$ for any a_A (recall that $\tilde{H}_A^1(a_A) \succeq_1 \tilde{H}_A^2(a_A)$) and, moreover,

$$\int_{a_{\max,2}^A}^{a_{\max,1}^A} \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_A)\right) d\tilde{H}_A^1(a_A) > \left(s(a_H^*) - s(a_H^* - (1-\beta)\, a_{\max,2}^A)\right) \left(1 - \tilde{H}_A^1(a_{\max,2}^A)\right).$$

Similarly, we can show that the second term in (30) evaluated at the non-discrimination quota:

$$SE_2|_{\theta=\theta^*} = -q^L \sum_i \gamma_i \frac{\partial a_{iL}^*}{\partial \mu} \int_0^{a_{\max,i}^A} \left(s(a_L^* - a_A + \beta a_A) - s(a_L^*)\right) h_N(a_L^* - a_A) dH_A^i(a_A),$$

is negative as well (the proof is exactly the same as that for SE_1). As a result, we can show that the selection effect evaluated at the non-discrimination quotas is negative, suggesting that we need to give quotas to the disadvantaged group. Moreover, if the groups are symmetric, then the selection effect is equal to zero at the non-discrimination quotas.